

Fibre spaces and Higher Categories

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Abstract

In this article we shall show another proof of Iitaka-Viehweg conjecture([I],[Vieh]), on some part of which is the way that Mochizuki's theory is applied. Secondly([Mch]), we consider global manifolds and a surjective morphism over a scheme from a product manifold onto a manifold with the automorphism group of general generic fibre algebraic. We apply Poincare-Segal n-groupoid to the situation above to prove that the latter manifold is also a product after a certain base-change([S3]).

1 Introduction

In the first section we show a revised version proof of Iitaka-Viehweg conjecture. To show Iitaka-Viehweg conjecture ([I],[Vieh]) $\max_{m \geq 1} \kappa(\det f_* \omega_{X/S}^{\otimes m}) \geq \text{var}(X/S)$ if the general generic fibre of X/S is of Kodaira dimension ≥ 0 , one constructs a cyclic covering $Y \rightarrow X$ such that the general generic fibre of Y/S is of general type and $\max_{m \geq 1} \kappa(\det g_* \omega_{Y/S}^{\otimes m}) \geq \text{var}(Y/S)$. One shall show $\text{var}(Y/S) \geq \text{var}(X/S)$ and that $\max_{m \geq 1} \kappa(\det f_* \omega_{X/S}^{\otimes m}) = \max_{m \geq 1} \kappa(\det g_* \omega_{Y/S}^{\otimes m})$. In the second section we briefly explain Poincare-Segal groupoids to apply it to a kind of deformation theory of manifolds with additional property([S3],[SGA]).

2 Iitaka-Viehweg Conjecture

2.1 Application of Mochizuki's Theory

Lemma 2.1. *Let k be a sub- p -adic field. Let S be a variety over k ([Mch]). Let U be a geometrically irreducible and reduced scheme surjective over S . Assume that U is trivial over S , i.e., there exists a variety U_0 such that U is isomorphic to $U_0 \times_k S$ over S . Let Γ_U denote the absolute Galois group of the rational function field of U (resp. Γ_S that of S , resp. Γ_k that of k). Then one obtains the next natural representation*

$$\rho : \Gamma_S \rightarrow \text{Aut}_{\Gamma_S}(\Gamma_U)$$

whose representation becomes trivial, i.e., one associates an identity homomorphism between Γ_U over Γ_S .

Proof. To each $g \in \Gamma_S$ one has an automorphism of Γ_U fixing Γ_S such that one associates $(u_0, s)^g = (u_0^g, s^g) = (u_0, gsg^{-1}) \in \Gamma_U$ to $(u_0, s) \in \Gamma_U = \Gamma_{U_0} \times_{\Gamma_k} \Gamma_S$ and one has the unique homomorphism which factors the homo-

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morphism above on the pull-back through inner automorphism $s \rightarrow s^g$ of Γ_S which is Γ_U :

$$\begin{array}{ccc} & \Gamma_U & \\ & \searrow & \\ \Gamma_U & \longrightarrow & \Gamma_U \\ \downarrow & & \downarrow \\ \Gamma_S & \longrightarrow & \Gamma_S \end{array}$$

□

$$\Gamma_X = \sqcup_{i \in I} x_i \Gamma_U$$

$$x_i u_i \mapsto x_i^g u_i^g$$

One constructs a new representation of Γ_S to a topological automorphism group $\text{Aut}_{\Gamma_S}(\Gamma_X)$ from a usual representation Γ_S to a topological automorphism group $\text{Aut}_{\Gamma_k}(\Gamma_X)$ when one has a continuous homomorphic section of Γ_U/Γ_S . One has a finite representatives $\{x_i\}_I$ in Γ_X such that there exists a cosets decomposition $\Gamma_X = \sqcup_{i \in I} x_i \Gamma_U$

$$x_i u_i \mapsto x_i^g u_i^g = x_{g(i)} u_{ig}$$

where there exists $u_{ig} \in \Gamma_S$ such that the equality above holds.

Let $\sigma : \Gamma_S \rightarrow \Gamma_U \subset \Gamma_X$ be a a continuous homomorphic section of $\pi : \Gamma_X \rightarrow \Gamma_S$. Let $g \in \Gamma_S$ and denote $\sigma(g)$ by g . A usual representation Γ_S to a topological automorphism group $\text{Aut}_{\Gamma_k}(\Gamma_X)$ when one has a continuous homomorphic section of Γ_U/Γ_S :

$$x_i u_i \mapsto x_i^g u_i^g$$

where $x_i^g = \sigma(g) x_i \sigma(g^{-1})$ and $u_i^g = \sigma(g) u_i \sigma(g^{-1})$. One there denotes $u_i = (u_{0i}, s_i) \in \Gamma_U = \Gamma_{U_0} \times_{\Gamma_k} \Gamma_S$. One finds $u_{ig} = (u_{0ig}, s_{ig}) \in \Gamma_U$ such that

$$x_i^g u_i^g = x_{g(i)} u_{ig}$$

where $i \mapsto g(i)$ is a member of a representation of Γ_S to a permutation group of $I : \Gamma_S \rightarrow \text{Sym}(I)$. Now one constructs two types of a new representation of Γ_S to a topological automorphism group $\text{Aut}_{\Gamma_S}(\Gamma_X)$:

1. Recall that $\pi : \Gamma_X \rightarrow \Gamma_S$ and σ its continuous homomorphic section and $u_i = (u_{0i}, s_i) \in \Gamma_U = \Gamma_{U_0} \times_{\Gamma_k} \Gamma_S$. Let $s'_{ig} \in \Gamma_S$ such that

$$\pi(x_i u_i) = \pi(x_i) s_i = \pi(x_{g(i)}) s'_{ig}$$

The map

$$x_i u_i \mapsto x_{g(i)} (u_{0ig}, s'_{ig})$$

gives a representation of Γ_S to $\text{Aut}_{\Gamma_S}(\Gamma_X)$.

2. Recall $x_i^g u_i^g = x_{g(i)} u_{ig}$ and denote $\pi(u_{ig}) = s_{ig}$. Let $s''_{ig} \in \Gamma_S$ such that

$$\pi(x_i) s''_i = \pi(x_{g(i)}) s_{ig} = \pi(x_i^g u_i^g)$$

The map

$$x_i (u_{0i}, s''_i) \mapsto x_i^g u_i^g = x_{g(i)} u_{ig} = x_{g(i)} (u_{0ig}, s_{ig})$$

gives a representation of Γ_S to $\text{Aut}_{\Gamma_S}(\Gamma_X)$.

Note that I is bijective to Γ_X/Γ_U as sets and it is a finite set by the hypothesis. Since $\text{Sym}(I)$ is a finite group, the cokernel of the representation $\Gamma_S \rightarrow \text{Sym}(I)$ is finite. Hence there exists a generically finite morphism $S' \rightarrow S$ such that $\Gamma_{S'} \cong \ker(\Gamma_S \rightarrow \text{Sym}(I))$. Note that the pull-back Γ_U along the inner automorphism $\Gamma_S \rightarrow \Gamma_S : x \mapsto gxg^{-1} = x^g$ is itself Γ_U and it denotes $\Gamma_U^{(g)}$. Note that $\Gamma_U = \Gamma_{U_0} \times_{\Gamma_k} \Gamma_S$. One obtains the following commutative diagram:

$$\begin{array}{ccccc} \Gamma_X & & & & \\ \downarrow & \searrow & & & \\ \Gamma_S & & \Gamma_X^{(g)} & \xrightarrow{\quad} & \Gamma_X \\ & & \downarrow & \searrow & \downarrow \\ & & \Gamma_S & \xrightarrow{\text{Inn}(g)} & \Gamma_S \end{array}$$

where the right side square is a pull-back and the bottom horizontal arrow is an inner automorphism by $g \in \Gamma_S$. Then one obtains the relative homomorphism $\tilde{g} : \Gamma_X \rightarrow \Gamma_X^{(g)} = \Gamma_X$ fixing Γ_S .

Thus one obtains

$$\Gamma_S \rightarrow \text{Aut}_{\Gamma_S}(\Gamma_X)(g \mapsto \tilde{g})$$

One can apply Mochizuki's theory ([Mch]).

Lemma 2.2. *From Mochizuki's Theorem ([Mch])*

$$\text{Aut}_{\Gamma_S}^{\text{open}}(\Gamma_X) / \ker(\Gamma_X \rightarrow \Gamma_S) \cong \text{Aut}_{k(S)}^{\text{opp}}(k(X))$$

one obtains

$$\Gamma_S \rightarrow \text{Aut}_{\Gamma_S}^{\text{open}}(\Gamma_X) / \ker(\Gamma_X \rightarrow \Gamma_S) \cong \text{Aut}_{k(S)}^{\text{opp}}(k(X)) \rightarrow \text{Out}(\Gamma_{X \times_S \overline{k(S)}}$$

Note that

$$\text{Out}(\Gamma_{X \times_S \overline{k(S)}}) \cong \text{Aut}(\Gamma_{X \times_S \overline{k(S)}}) / \ker(\Gamma_{X \times_S \overline{k(S)}} \rightarrow \Gamma_{\overline{k(S)}})$$

The representation $\Gamma_S \rightarrow \text{Out}(\Gamma_{X \times_S \overline{k(S)}})$ factors through the algebraic group scheme which is the birational automorphism group of $X \times_S \overline{k(S)}$ ([Mat], [RBZL]). One can take a subgroup $\Gamma_{S''}$ of finite index of $\Gamma_{S'}$, such that $\Gamma_{S''}$ just maps to a neutral component of the birational automorphism group $\text{Bir}^o(X \times_S \overline{k(S)})$.

Since one has the next isomorphism

$$H^1(\Gamma_S, \text{Bir}^o(X \times_S k(S))) \cong H_{et}^1(\overline{k(S)}/k(S), \text{Bir}^o(X \times_S k(S)))$$

One can find a generically finite morphism $S' \rightarrow S$ such that a torsor with group action by $\text{Bir}^o(X \times_S k(S))$ over $\text{Spec}(k(S))$ associated to an element of $H^1(\Gamma_S, \text{Bir}^o(X \times_S k(S)))$ becomes a trivial torsor, that is a neutral element of $H^1(\Gamma_{S'}, \text{Bir}^o(X \times_S k(S')))$ since a torsor with group action by $\text{Bir}^o(X \times_S k(S'))$ over $\text{Spec}(k(S'))$ can have a section. Therefore, the element $H^1(\Gamma_{S'}, \text{Out}(\Gamma_{X \times_S \overline{k(S)}}))$ is trivial which is an image of the element

$$H^1(\Gamma_{S'}, (\Gamma_{X \times_S \overline{k(S)}}) \rightarrow \text{Aut}(\Gamma_{X \times_S \overline{k(S)}})$$

representing the extension

$$\Gamma_{X \times_S \overline{k(S)}} \rightarrow \Gamma_X \rightarrow \Gamma_S$$

([AM], [BJ], [Breen1], [Breen2], [Gir], [GG]).

2.2 Weak Positivity

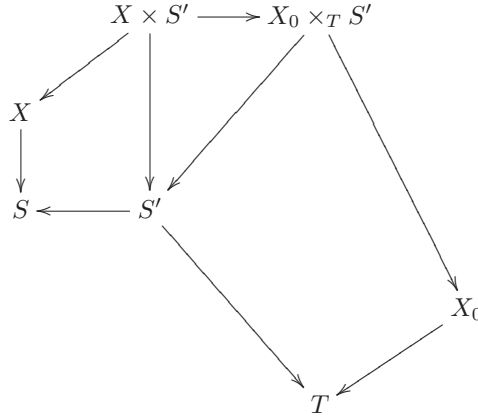
Our main aim was to show the following theorem.

Theorem 2.1. *Let $f : X \rightarrow S$ be a fibre space of non singular varieties. Assume that $\kappa(\omega_{X_{\bar{\eta}}}) \geq 0$ for the generic geometric fibre $X_{\bar{\eta}}$. Then there exists an integer $m > 0$ such that $\kappa(\det f_* \omega_{X/S}^{\otimes m}) \geq \text{var}(X/S)$.*

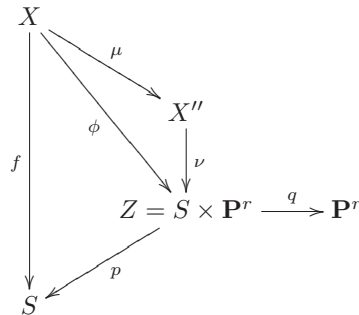
Here $\det f_* \omega_{X/S}^{\otimes m}$ is taken to be a divisorial sheaf. Viehweg's Lemma:

Lemma 2.3. *Let $S' \rightarrow S$ be a Kawamata covering with respect to an ample divisor. Let $X \times_S S' \rightarrow S'$ be the pull-back. Then $X \times_S S'$ has rational singularity. Further take a desingularization $X^! \rightarrow X \times_S S'$. Then $\kappa(\det f_*^! \omega_{X^!/S'}^{\otimes m}) \leq \kappa(\det f_* \omega_{X/S}^{\otimes m})$.*

We shall prove the theorem above in the following several steps. By Viehweg in order to prove the theorem, we can assume further that $\text{var}(X/S) = \dim S$ and show the theorem.



Let $f : X \rightarrow S$ be a fibre space. From the extension of the function fields $R(X)/R(S)$, we have purely transcendental indeterminates t_1, \dots, t_r over $R(S)$ such that $R(X)/R(S)(t_1, \dots, t_r)$ is a finite extension of degree d . Hence we obtain a dominant rational map $X \rightarrow S \times \mathbf{P}^r$. Resolving the indeterminacy of the rational map $X \rightarrow S \times \mathbf{P}^r$, we have a birational map $X' \rightarrow X$ and a morphism $\phi : X' \rightarrow S \times \mathbf{P}^r$. We replace X' by X and let Z denote $S \times \mathbf{P}^r$. Let X'' be the integral closure of Z in the function field $R(X)$. Namely, $\mu : X \rightarrow X''$ with $\nu : X'' \rightarrow Z$ is Stein factorization. Let $\mu : X \rightarrow X''$ be the structure morphism.



Recall the next lemma due to Kawamata-Viehweg([Kaw],[Vieh],[Fuj]).

Lemma 2.4. $\omega_{X/S}$ is weakly positive with respect to f .

In other words, given any $\alpha > 0$ and any big \mathbf{Q} -invertible sheaf L over S , it holds that $\kappa(\omega_{X/S}^{\otimes \alpha} \otimes f^*L) \geq \dim S$.

We have a rational map $X \rightarrow \mathbf{P}(\Gamma(X, (\omega_{X/S} \otimes f^*L)^{\otimes m}))$ defined by $\mathcal{O}_X \otimes \Gamma(X, (\omega_{X/S} \otimes f^*L)^{\otimes m}) \rightarrow (\omega_{X/S} \otimes f^*L)^{\otimes m}$ for $m \gg 0$. Take a resolution of the indeterminacy of the rational map, which is denoted by $X^* \rightarrow X$. Then replace X^* by X . Since X is non singular, there exists an effective \mathbf{Q} -cartier divisor D such that $\omega_{X/S}^{\otimes \alpha} \otimes f^*L = \mathcal{O}_X(D)$ for any $\alpha > 0$.

Lemma 2.5. *Let D be an effective \mathbf{Q} -divisor on X . There exist an effective \mathbf{Q} -divisor E and an effective Weil divisor D' on X'' $D = \mu^*D' + E$ such that E is a μ -exceptional divisor, i.e., the μ image of the support of E in X'' is of codimension ≤ 2 .*

Proof. Since $\mu : X \rightarrow X''$ is birational, there exists a locus of codimension ≤ 2 outside which the restriction of μ is an isomorphism. Hence we have an effective \mathbf{Q} -divisor decomposition $D = \mu^*D' + E$ such that E is a μ -exceptional divisor. \square

Lemma 2.6. *There exist \mathbf{Q} -ample divisors C_1 and C_2 such that $D' = C_1 - C_2$ in $Z^{(1)}(X'' \otimes \mathbf{Q})$ up to \mathbf{Q} -linear equivalence.*

Proof. There exists a \mathbf{Q} -ample divisor C_1 such that $C_1 - D'$ is \mathbf{Q} -ample, say, C_2 . \square

Lemma 2.7. *There exist \mathbf{Q} -ample divisors D_1 and D_2 over Z such that $C_1 = \nu^*D_1$ and $C_2 = \nu^*D_2$ up to \mathbf{Q} -linear equivalence.*

Proof. We refer to the next lemma.

Lemma 2.8 (EGA4-3). *Let $f : X \rightarrow Y$ be a proper flat morphism of finite presentation. The set of $y \in Y$ such that X_y is smooth over $k(y)$ is open.*

Since $f : X \rightarrow S$ is a fibre space of non singular varieties, i.e., a projective connected morphism, a general fibre X_s for a closed point $s \in S$ is a non singular variety. Let Div_S be a scheme representing a functor $T \rightarrow \text{Div}_{S/k}(T)$. Its components are quasi-projective. Let Γ be the universal relative effective divisor on $S \times \text{Div}_S / \text{Div}_S$. Note that a fibre of a closed point of S for $\Gamma \subset S \times \text{Div}_S \rightarrow S$ is an effective divisor on S . A general fibre of a closed point $s \in S$ for $f^{-1}\Gamma \subset X \times \text{Div}_S \rightarrow S \times \text{Div}_S \rightarrow \text{Div}_S$ is a non singular variety, i.e., smooth and irreducible over $k(s)$. Let Ξ be the universal relative effective divisor on $Z \times \text{Div}_Z$. Since the pullback of the universal relative effective divisor over $\Gamma \subset S \times \text{Div}_S$ to $p^{-1}\Gamma \subset Z \times \text{Div}_S$ gives a morphism $\text{Div}_S \rightarrow \text{Div}_Z$, a general fibre of a closed point t of Div_Z for $\phi^{-1}\Xi \subset X \times \text{Div}_Z \rightarrow \text{Div}_Z$ is a non singular variety. Hence there exists a dense open set U of Div_Z such that $\phi^{-1}\Xi|_U$ is smooth and irreducible. Therefore there exists one point in X'' over the generic point of an effective divisor on Z corresponding to a closed point $t \in U$.

A general member of the linear system of a sufficiently ample divisor on X'' is irreducible and moves freely. Hence the direct image of a general member by ν is able to be a general irreducible divisor on Z , which is associated with a closed point $t \in U$.

Hence there exist \mathbf{Q} -ample divisors D_1 and D_2 over Z such that $C_1 = \nu^*D_1$ and $C_2 = \nu^*D_2$ up to \mathbf{Q} -linear equivalence. \square

Note that $\text{Pic}(Z) = \text{Pic}(S) \times \text{Pic}(\mathbf{P}^r)$ and that $\text{Pic}(\mathbf{P}^r) \cong \mathbf{Z}$. Let $p : Z \rightarrow S$ and $q : Z \rightarrow \mathbf{P}^r$. Let $\phi = \nu \circ \mu : X \rightarrow Z$. Now put them together. We have

1. $\omega_{X/S} = \mathcal{O}_X(D - f^*L)$

2. $D = \mu^* D' + E$
3. $D' = C_1 - C_2 = \nu^*(D_1 - D_2)$
4. $D_i = p^* A_i + a_i q^* H$, where A_i is an ample \mathbf{Q} -divisor on S , H is a hyperplane section of \mathbf{P}^r , a_i is a rational number and $i = 1, 2$.
5. $\omega_{X/S} = \mathcal{O}_X(-f^* L + E + \phi^*(p^* A_1 + a_1 q^* H - p^* A_2 - a_2 q^* H)) = \mathcal{O}_X(E + f^* p^* A + b q^* H)$, where $A = -L + A_1 - A_2$, $a = a_1 - a_2$.
6. $\mu_* \omega_{X/S}^{\otimes m} = \mu_* \mathcal{O}_X(m(E + f^* p^* A + a q^* H)) = \mu_* \mathcal{O}_X(m(f^* p^* A + a q^* H))$.

Note that $\phi_* \mathcal{O}_X(mE) = \phi_* \mathcal{O}_X$.

Lemma 2.9. $\phi_* \mathcal{O}_X \subset \oplus^d \mathcal{O}_Z$

Proof. We apply the next lemma to a finite morphism X''/Z . We refer to Weierstrass preparation lemma.

Lemma 2.10. *Let a convergent series $g \in k\{x_1, \dots, x_n\}$ such that $g(0, \dots, x_n) \neq 0$ and order p . Then $B = k\{x_1, \dots, x_n\}/(g)$ is a free module over the ring $A = k\{x_1, \dots, x_{n-1}\}$ and has a basis of classes $(1, x_n, \dots, x_n^{p-1}) \bmod (g)$.*

The convergent series rings are strictly henselian([SGA]). Hence X''/Z is Kummer gerbe, i.e., cyclic cover, with respect to the étale topology. We have $\nu_* \mathcal{O}_{X''} = \oplus_{i=0}^{d-1} \mathcal{O}_Z(-iD)$, where D is an effective divisor with respect to the étale topology. Note, however, that the composition of the morphisms $X'' \rightarrow Z$ and $Z \rightarrow \mathbf{P}^r$ is a connected morphism. □

2.3 Cyclic Covering

Let $L = \mathcal{O}_X(\phi^* q^*(bH))$. Here $b > 0$ is taken sufficiently large. Choose a non singular irreducible divisor D such that $L^{\otimes m} = \mathcal{O}_X(D)$. Take a cyclic cover $Y = \text{Spec } \oplus_{0 \leq i \leq n-1} L^{\otimes i}$ of X , which denotes $\tau : Y \rightarrow X$. Note that Y is a non singular variety and Y/S is a fibre space. By adjunction formula, $\mathcal{O}_Y(K_Y) = \mathcal{O}_Y(\tau^* K_X \otimes \tau^* L^{\otimes(n-1)})$. It is well known $\tau_* \mathcal{O}_Y = \oplus_{0 \leq i \leq n-1} (L^{-1})^{\otimes i}$. Hence by projection formula, $\tau_* \omega_{Y/S}^{\otimes m} = \tau_* \mathcal{O}_Y \otimes_{\mathcal{O}_X} \mathcal{O}_X(\omega_{X/S}^{\otimes m}) \otimes_{\mathcal{O}_X} L^{\otimes m(n-1)}$. Let $g = f \circ \tau$. One obtains $g_* \omega_{Y/S}^{\otimes m} \subset \oplus_{0 \leq i \leq n-1} f_* \omega_{X/S}^{\otimes m} \otimes L^{\otimes(m(n-1)-i)}$ and $\det g_* \omega_{Y/S}^{\otimes m} \subset \otimes^d \oplus_{0 \leq i \leq n-1} \det f_* \omega_{X/S}^{\otimes m} \otimes_{\mathcal{O}_X} L^{\otimes(m(n-1)-i)}$.

Proposition 2.1. $g_* \omega_{Y/S}^{\otimes m} = f_*(\tau_* \mathcal{O}_Y \otimes \mathcal{O}_X(\omega_{X/S}^{\otimes m})) \otimes L^{\otimes m(n-1)} \subset \oplus^d \oplus_{0 \leq i \leq n-1} \oplus \mathcal{O}_S^{r_i} \otimes \mathcal{O}_S(mA)$, where $r_i = \dim \Gamma(\mathbf{P}^r, \mathcal{O}((ma + b(m(n-1) - i))H))$

Proof. From the argument above, we have $g_* \omega_{Y/S}^{\otimes m} \subset \oplus_{0 \leq i \leq n-1} f_* \omega_{X/Y}^{\otimes m} \otimes L^{\otimes(m(n-1)-i)} \subset \oplus_{0 \leq i \leq n-1} p_* \phi_* \mathcal{O}_X(mE) \otimes \mathcal{O}_Z((maq^* H + mp^* A + b(m(n-1) - i))H) \subset \oplus^d \oplus_{0 \leq i \leq n-1} p_* \mathcal{O}_Z((ma + b(m(n-1) - i))q^* H) \otimes p^* \mathcal{O}_S(mA) = \oplus^d \oplus_{0 \leq i \leq n-1} \oplus \mathcal{O}_S^{r_i} \otimes \mathcal{O}_S(mA)$. Here $r_i = \dim \Gamma(\mathbf{P}^r, \mathcal{O}((ma + b(m(n-1) - i))H))$. See the following lemma.

Lemma 2.11. $p_* \mathcal{O}_Z((ma + b(m(n-1) - i))q^* H) = \oplus \mathcal{O}_S^{r_i}$, where $r_i = \dim \Gamma(\mathbf{P}^r, \mathcal{O}((ma + b(m(n-1) - i))H))$. □

We may assume that if b is taken sufficiently large, the generic geometric fibre of Y/S is of general type, if necessary, the canonical invertible sheaf over the generic geometric fibre of Y/S is abundant with $R^i g_* \omega_{Y/S}^{\otimes m} = 0$ for $i > 0$. The composite map $\tau \circ \nu \circ \mu : Y \rightarrow Z$ is generically finite and the invertible sheaf $q^* H$ is relatively ample with respect to $p : Z \rightarrow S$.

Lemma 2.12. *Let S be a non singular variety. Let L be an invertible sheaf and E' and E locally free sheaves of finite rank over S . Given the exact sequence $0 \rightarrow E' \rightarrow E \otimes L$ and $E \cong \mathcal{O}^n$ for some $n > 0$, then $\kappa(L^{\otimes r} \otimes (\det E')^{-1}) \geq 0$, where $r = \text{rank } E'$.*

Proof. Take the dual and we have a homomorphism $(E \otimes L)^* \rightarrow (E')^*$ and let the image be F and K the kernel. F and K are torsion free and locally free outside a closed subset of codimension ≥ 2 , which we denote S° . F is of the same rank as E' . We have the exact sequences $0 \rightarrow (E \otimes L)^* \rightarrow F \rightarrow 0$ and $0 \rightarrow K \otimes L \rightarrow (E')^* \rightarrow F \otimes L \rightarrow 0$ over S° . Thus $F \otimes L$ is globally generated and $F \otimes L \rightarrow (E')^* \otimes L$ is an isomorphism over S° . Hence $\det(F \otimes L) \subset \det((E')^* \otimes L)$. Note that $\det((E')^* \otimes L) = L^{\otimes r} \otimes (\det E')^{-1}$, where $r = \text{rank } E'$. Therefore $\kappa(L^{\otimes r} \otimes (\det E')^{-1}) \geq 0$. \square

Proposition 2.2. $\kappa(\mathcal{O}_S(mA)) \geq \kappa(\det g_* \omega_{Y/S}^{\otimes m})$

Proof. Apply the lemma above to the following formula, $g_* \omega_{Y/S}^{\otimes m} \subset \bigoplus^d \bigoplus_{0 \leq i \leq n-1} \bigoplus \mathcal{O}_S^{r_i} \otimes \mathcal{O}_S(mA)$, where $r_i = \dim \Gamma(\mathbf{P}^r, \mathcal{O}((ma + b(m(n-1) - i))H))$. \square

Consider the case when $m = 1$. Let D be a classifying space for a variation of Hodge structure and let Γ be the monodromy group, which is a subgroup of the arithmetic group of all linear automorphism group of $H^{\dim X_s}(X_s, \mathbf{C})$ which preserve a certain condition. Let $\Phi : S \rightarrow \Gamma \backslash D$ be a holomorphic period mapping satisfying the Griffiths' transversality relation. A period mapping Φ gives rise to a variation of Hodge structure by pulling back the universal family over $\Gamma \backslash D$. Since the generic geometric fibre of Y/S is of general type and $\text{var}(Y/S) \geq \dim S$, the period mapping Φ is a finite to one mapping. Hence we obtain $\kappa(A) \dim S([\text{Ws}])$.

One obtains $g_* \omega_{Y/S}^{\otimes m} \subset \bigoplus_{0 \leq i \leq n-1} f_* \omega_{X/S}^{\otimes m} \otimes L^{\otimes(m(n-1)-i)}$ and

$$\det g_* \omega_{Y/S}^{\otimes m} \subset \bigotimes^d \bigoplus_{0 \leq i \leq n-1} \det f_* \omega_{X/S}^{\otimes m} \otimes_{\mathcal{O}_X} L^{\otimes(m(n-1)-i)}$$

. Consider the following homomorphism

$$\theta : p^* \bigotimes^d \bigoplus_{0 \leq i \leq n-1} \det f_* \omega_{X/S}^{\otimes m} \otimes_{\mathcal{O}_X} \mathcal{O}_X^{\otimes(m(n-1)-i)} \rightarrow p^* \bigotimes^d \bigoplus_{0 \leq i \leq n-1} \det f_* \omega_{X/S}^{\otimes m} \otimes_{\mathcal{O}_X} L^{\otimes(m(n-1)-i)}$$

- The θ is isomorphic on the $Z \setminus q^{-1} \text{support } H$.
- The θ is isomorphic on the $Z \setminus p^{-1} \text{support } \bigotimes^d \bigoplus_{0 \leq i \leq n-1} \det f_* \omega_{X/S}^{\otimes m} \otimes_{\mathcal{O}_X} L^{\otimes(m(n-1)-i)}$.
- Hence θ is isomorphic on the whole Z .

•

$$\kappa(\det g_* \omega_{Y/S}^{\otimes m}) \leq \kappa(\det f_* \omega_{X/S}^{\otimes m} \otimes_{\mathcal{O}_X} L^{\otimes(m(n-1)-i)})$$

•

$$\kappa(\det f_* \omega_{X/S}^{\otimes m} \otimes_{\mathcal{O}_X} L^{\otimes(m(n-1)-i)}) = \kappa(\det f_* \omega_{X/S}^{\otimes m} \otimes_{\mathcal{O}_X} \mathcal{O}_X^{\otimes(m(n-1)-i)})$$

•

$$\kappa(\det g_* \omega_{Y/S}^{\otimes m}) \leq \kappa(\det f_* \omega_{X/S}^{\otimes m} \otimes_{\mathcal{O}_X} \mathcal{O}_X^{\otimes(m(n-1)-i)}) = \kappa(\det f_* \omega_{X/S}^{\otimes m})$$

This implies

$$\text{var}(X/S) \leq \text{var}(Y/S) = \kappa(\det g_* \omega_{Y/S}^{\otimes m}) \leq \kappa(\det f_* \omega_{X/S}^{\otimes m})$$

We therefore show the revised version of Iitaka-Viehweg Conjecture([Kaw],[Vieh],[I]).

3 Poincare-Segal n -Groupoids

In this section we briefly introduce the notion of Poincare-Segal n -groupoids which we refer to Carlos Simpson's Homotopy Theory of Higher Categories([S3]).

Recall that a groupoid is a category where the morphisms are all invertible.

Definition 3.1. *The following definitions are given by mutual induction on $n \geq 1$:*

- *a strict n -category \mathcal{A} is a strict n -groupoid if for all $x, y \in \mathcal{A}$, $\mathcal{A}(x, y)$ is a strict $(n-1)$ -groupoid and for any 1-morphism $u : x \rightarrow y$ in \mathcal{A} the two morphism of composition with u*

$$\mathcal{A}(y, z) \rightarrow \mathcal{A}(x, z), \quad \mathcal{A}(w, x) \rightarrow \mathcal{A}(w, y)$$

are equivalences of strict $(n-1)$ -groupoids;

- *a morphism $f : \mathcal{A} \rightarrow \mathcal{B}$ between n -groupoids is essentially surjective if for every $x \in \text{Ob}(\mathcal{B})$ there exists $z \in \text{Ob}(\mathcal{A})$ and an arrow $u \in \text{Ob}(\mathcal{B}(z, f(x)))$ is nonempty;*
- *a morphism $f : \mathcal{A} \rightarrow \mathcal{B}$ between n -groupoids is fully faithful if for all $x, y \in \text{Ob}(\mathcal{A})$ the map $\mathcal{A}(x, y) \rightarrow \mathcal{B}(f(x), f(y))$ is an equivalence of $(n-1)$ -groupoids; and*
- *a morphism $f : \mathcal{A} \rightarrow \mathcal{B}$ between n -groupoids is an equivalence if it is fully faithful and essentially surjective.*

Theorem 3.1. ([S3]) *A strict n -category \mathcal{A} is an n -groupoid if and only if for $x, y \in \text{Ob}(\mathcal{A})$ the $(n-1)$ -category of morphisms $\mathcal{A}(x, y)$ is an $(n-1)$ -groupoid.*

If \mathcal{A} is an n -groupoid, $\pi_0(\mathcal{A})$ (resp. $\tau_{\leq 0}(\mathcal{A})$) is defined to be the quotient of $\text{Ob}(\mathcal{A})$ by the relation of inner equivalence. A truncated 1-category $\tau_{\leq 0}(\mathcal{A})$ is defined to be

$$\text{Ob}(\tau_{\leq 1}(\mathcal{A})) = \text{Ob}(\mathcal{A})$$

$$(\tau_{\leq 1}(\mathcal{A}))(x, y) = \pi_0(\mathcal{A}(x, y)).$$

Theorem 3.2. ([S3]) *If \mathcal{A} is an n -groupoid, for any $0 \leq k \leq n$ there exists a strict k -groupoid $\tau_{\leq k}\mathcal{A}$ the i -morphism of which are those of \mathcal{A} for $i < k$ and k -morphisms of which are the equivalence classes of k -morphisms of \mathcal{A} under the equivalence relation of inner equivalence. There exists the natural projection $\mathcal{A} \rightarrow \tau_{\leq k}\mathcal{A}$ which is a morphism of n -categories.*

Let nSTRGPD be the category of strict n -groupoids and Top the category of topological spaces.

Definition 3.2. *A realization functor for strict n -groupoids is a functor*

$$\mathcal{R} : \text{nSTRGPD} \rightarrow \text{Top}$$

together with the following natural transformation:

$$r : \text{Ob}(\mathcal{A}) \rightarrow \mathcal{R}(\mathcal{A})$$

$$\zeta_i(\mathcal{A}, x) : \pi_i(\mathcal{A}, x) \rightarrow \pi_i(\mathcal{R}(\mathcal{A}), r(x))$$

the latter including $\zeta_0(\mathcal{A}) : \pi_0(\mathcal{A}) \rightarrow \pi_0(\mathcal{R}(\mathcal{A}))$ such that $\zeta_i(\mathcal{A}, x)$ and $\zeta_0(\mathcal{A})$ are isomorphisms for $0 \leq i \leq n$ such that ζ_0 takes the isomorphism class of x to the connected component of $r(x)$ and such that the $\pi_i(\mathcal{R}(\mathcal{A}), y)$ vanish for $i > n$.

Theorem 3.3. ([S3]) *There exists a realization functor \mathcal{R} for strict n -groupoids.*

Let \mathcal{M} be a category and X a set.

Definition 3.3. *An \mathcal{M} -precategory over a set X is a functor $\mathcal{F} : \Delta_X^\circ \rightarrow \mathcal{M}$ such that $\mathcal{F}(x_0) \cong *$ is the coinitial object of \mathcal{M} for any $x_0 \in X$. Let $\mathcal{PC}(X, \mathcal{M})$ denote the category of \mathcal{M} -precategories over X with morphisms which are natural transformations of diagrams.*

$$\mathcal{PC}(X, \mathcal{M}) = \text{Func}(\Delta_X^\circ / X, \mathcal{M})$$

One says that an \mathcal{M} -precategory satisfies Segal conditions if the Segal maps are weak equivalences $\mathcal{W} \subset \mathcal{M}$: the Segal maps at (x_0, \dots, x_n)

$$\mathcal{A}(x_0, \dots, x_n) \rightarrow \mathcal{A}(x_0, x_1) \times \dots \times \mathcal{A}(x_{n-1}, x_n).$$

When $\mathcal{M} = \mathcal{K}$ is Kan-Quillen model category of simplicial sets, it is a tractable left proper cartesian model category. A \mathcal{K} -precategory is called a Segal precategory and if it satisfies Segal conditions it is called a Segal category. When \mathcal{K} is the category of presheaves on Δ , one has a natural identification

$$\mathcal{PC}(\mathcal{K}) \cong \text{Presheaf}(\mathbf{Cone}(\Delta)) \subset \text{Presheaf}(\Delta \times \Delta)$$

A Segal precategory is a pair (X, \mathcal{A}) , where $X \in \text{SET}$ and \mathcal{A} is a collection of simplicial sets $\mathcal{A}(x_0, \dots, x_n)$ for sequences of $x_i \in X$. Note that $\mathbf{Cone}(\Delta)$ is a quotient of $\Delta \times \Delta$.

Definition 3.4. *A Segal groupoid is a Segal category such that $\tau_{\leq 1}(\mathcal{A})$ is a groupoid.*

Given a fibrant object X in \mathcal{K} , i.e., a Kan simplicial set, one can define the Poincare-Segal groupoid $\Pi_S(X)$, which is a Segal groupoid. It is constructed as the right adjoint of the diagonal realization functor. The diagonal $d : \Delta \rightarrow \Delta \times \Delta$ equips a pullback functor

$$d^* : \text{Presheaf}(\Delta \times \Delta) \rightarrow \text{Presheaf}(\Delta) = \mathcal{K}$$

composing the functor above one obtains the realization function

$$| \cdot | : \mathcal{PC}(\mathcal{K}) \rightarrow \mathcal{K}$$

Theorem 3.4. ([S3]) *If X is a fibrant simplicial set then $\Pi_S(X)$ is fibrant, so it is a Segal category. It is in fact a Segal groupoid and the counit of the adjunction*

$$|\Pi_S(X)| \rightarrow X$$

is a weak equivalence. Conversely, if $\mathcal{A} \in \mathcal{PC}(\mathcal{K})$ is a Segal groupoid and $|\mathcal{A}| \rightarrow Y$ is a fibrant replacement in \mathcal{K} , then the adjunction

$$\mathcal{A} \rightarrow \Pi_S(Y)$$

is a global equivalence of Segal categories.

A morphism $f : \mathcal{A} \rightarrow \mathcal{B}$ between Segal groupoids is a global equivalence if and only if $\pi_0(\mathcal{A}) \rightarrow \pi_0(\mathcal{B})$ is surjective and for each $i \geq 1$ and each object $x \in \text{Ob}(\mathcal{A})$, the induced maps $\pi_i(\mathcal{A}, x) \rightarrow \pi_i(\mathcal{B}, f(x))$ are isomorphisms. If \mathcal{A} is a Segal groupoid then $\pi_0(\mathcal{A}) = \pi_0(|\mathcal{A}|)$ and for any $i \geq 1$ and $x \in \text{Ob}(\mathcal{A})$,

$$\pi_i(\mathcal{A}, x) = \pi_i(|\mathcal{A}|, x).$$

When \mathcal{M} is a tractable left proper cartesian model category, the model category $\mathcal{PC}_{Reedy}(\mathcal{M})$ of \mathcal{M} -enriched precategories with Reedy cofibrations and global weak equivalences is again a tractable left proper cartesian model category. Therefore one can iterate the construction to obtain various versions of model categories for n -categories and similar objects. For any $n \geq 0$ define by induction $\mathcal{PC}^0(\mathcal{M}) = \mathcal{M}$ and for $n \geq 1$

$$\mathcal{PC}(\mathcal{M}) = \mathcal{PC}(\mathcal{PC}^{n-1}(\mathcal{M})).$$

This is the model category of \mathcal{M} -enriched n -precategories.

Definition 3.5. An \mathcal{M} -enriched n -precategory $\mathcal{A} \in \mathcal{PC}(\mathcal{M})$ satisfies the full Segal condition if it satisfies the Segal condition as an $\mathcal{PC}^{n-1}(\mathcal{M})$ -precategory, and furthermore inductively for any sequence of objects $x_0, \dots, x_m \in \text{Ob}(\mathcal{A})$ the \mathcal{M} -enriched $(n-1)$ -precategory

$$\mathcal{A}(x_0, \dots, x_m) \in \mathcal{PC}^{n-1}(\mathcal{M})$$

satisfies the full Segal condition.

If \mathcal{A} is a fibrant object in the model structure on $\mathcal{PC}^n(\mathcal{M})$, then \mathcal{A} satisfies the full Segal condition.

Let $\mathcal{M} = \mathcal{K} = \text{Presheaf}(\Delta)$. One has

$$\mathcal{PC}^n(\mathcal{K}) = \text{Presheaf}(\text{Cone}^n(\Delta))$$

This is the model category of Segal n -precategories. If $\mathcal{M} = \text{Set}$, $\mathcal{PC}(\mathcal{M})^n$ is called the category of n -prenerves or n -precategories. An n -nerve or n -category is an n -prenerve \mathcal{A} satisfying the full Segal condition.

The Poincare-Segal groupoid functor

$$\Pi_S : \mathcal{K} \rightarrow \mathcal{PC}(\mathcal{K})$$

can be iterated to give a right Quillen functor

$$\Pi_{n,S} : \mathcal{K} \rightarrow \mathcal{PC}^n(\mathcal{K})$$

which is called the Poincare-Segal n -groupoid. The functor $\Pi_{n,S}$ has a left adjoint $|\cdot|$, which is the multidagonal realization on $(n+1)$ -simplicial sets.

Theorem 3.5. ([S3]) If $\mathcal{A} \rightarrow \mathcal{B}$ is a morphism of Segal n -groupoids, then it is a weak equivalence if and only if $|\mathcal{A}| \rightarrow |\mathcal{B}|$ is a weak equivalence. Suppose $X \in \mathcal{K}$ is a fibrant simplicial set. Then

$$|\Pi_{n,S}(X)| \rightarrow X$$

is a weak equivalence of simplicial sets. Conversely, if $\mathcal{A} \in \mathcal{PC}^n(\mathcal{K})$ is a fibrant Segal n -groupoid, then

$$\mathcal{A} \rightarrow \Pi_{n,S}(\mathbf{Ex}^\infty |\mathcal{A}|)$$

is a weak equivalence in $\mathcal{PC}^n(\mathcal{K})$. The pair of these functors provides an equivalence between

$$\text{ho}(\mathcal{K}) \cong \text{ho}(\text{Top})$$

and the homotopy category given by localizing the subcategory of Segal n -groupoids in $\mathcal{PC}^n(\mathcal{K})$ with weak equivalences.

([S3]) For n -nerves, the right Quillen functor

$$\Pi_n : \mathcal{K} \rightarrow \mathcal{PC}^n(\text{Set})$$

is the Poincare n -groupoid the left adjoint of which is denoted $|\cdot|$. The truncation functor $\tau_{\leq 0}$ is left adjoint to the inclusion $\text{Set} \rightarrow \mathcal{K}$ since sets are considered as discrete simplicial sets. Note that the truncation functor $\tau_{\leq 0} : \mathcal{K} \rightarrow \text{Set}$ induces

$$\tau_{\leq n} : \mathcal{PC}^n(\mathcal{K}) \rightarrow \mathcal{PC}^n(\text{Set})$$

and

$$\Pi_n(X) = \tau_{\leq n} \Pi_{n,S}(X)$$

Thus $\tau_{\leq n}$ is left adjoint to the following map

$$\mathcal{PC}^n(\text{Set}) \rightarrow \mathcal{PC}^n(\mathcal{K}) \rightarrow \mathcal{K}$$

This composition is the realization for n -groupoids which are n -nerves that satisfy the groupoid condition in all dimension. We finally refer to the following theorem about limits and colimits.

Theorem 3.6. ([S3]) *The $(n+1)$ -category $n\text{Cat}$ of all small n -categories, admits limits (resp. colimits) indexed by small $(n+1)$ -categories. The Segal $(n+1)$ -category $n\text{SeCat}$ admits limits (resp. colimits) indexed by small Segal $(n+1)$ -categories.*

3.1 Deformation of Differential manifolds

It is known that for each compact manifold M the diffeomorphism group $\text{Diff}(M)$ is a regular Lie group and that its Lie algebra is the space $\mathfrak{X}(M)$ of all smooth vector fields on M , with the negative of the usual bracket as Lie bracket.

Diffeomorphism groups of compact manifolds of larger dimension are regular Frechet Lie groups; very little about their structure is known.

For finite dimensional manifolds M and N with M compact, the space $\text{Emb}(M, N)$ of all smooth embeddings of M into N , is open in $C^\infty(M, N)$, so it is a smooth manifold. The diffeomorphism group $\text{Diff}(M)$ acts freely and smoothly from the right on $\text{Emb}(M, N)$.

Theorem 3.7. ([AC]) *$\text{Emb}(M, N) \rightarrow \text{Emb}(M, N)/\text{Diff}(M)$ is a principal fiber bundle with structure group $\text{Diff}(M)$.*

The identity component of any Lie group is an open normal subgroup, and the quotient group is a discrete group. The universal cover of any connected Lie group is a simply connected Lie group, and conversely any connected Lie group is a quotient of a simply connected Lie group by a discrete normal subgroup of the center. Any Lie group G can be decomposed into discrete, simple and abelian groups in a canonical way as follows. Write G_{con} for the connected component of the identity G_{sol} for the largest connected normal solvable subgroup G_{nil} for the largest connected normal nilpotent subgroup so that we have a sequence of normal subgroups

$$1 \triangleleft G_{\text{nil}} \triangleleft G_{\text{sol}} \triangleleft G_{\text{con}} \triangleleft G.$$

Then

- G/G_{con} is discrete

- G_{con}/G_{sol} is a central extension of a product of simple connected Lie groups.
- G_{sol}/G_{nil} is abelian. A connected abelian Lie group is isomorphic to a product of copies of \mathbb{R} and the circle group S_1 .
- $G_{nil}/1$ is nilpotent, and therefore its ascending central series has all quotients abelian.

The semi-simple complex Lie group is an algebraic group.

Theorem 3.8. *Let S be a scheme algebraic over a field k of characteristic 0. Let X and Y be stacks over S . Suppose that X is a trivial stack, i.e., X is isomorphic to a product $X_0 \times S$ where X_0 is defined over k and that there exists a strictly epimorphic morphism $f : X \rightarrow Y$ defined over S . Assume that the fibres of Y/S are differentiable manifolds (resp. with additional structure spectral triple (A, H, D) consisting of a representation of a C^* -algebra A on a Hilbert space H , together with an unbounded operator D on H , with compact resolvent, such that $[D, a]$ is bounded for all a in some dense subalgebra of A) and that the automorphism groups of the fibres if the base field is changed to an algebraically closed field \bar{k} are isomorphic to complex semi-simple Lie groups. Then Y is also trivial over S .*

Proof. Let $\Pi_{n,S}(X)$, $\Pi_{n,S}(Y)$ and $\Pi_{n,S}$ be the Poincare-Segal n -groupoids of X, Y and S , respectively. Here n is taken to be large enough. Then one has the following diagram

$$\begin{array}{ccc} \Pi_{n,S}X & \xrightarrow{\quad} & \Pi_{n,S}Y \\ & \searrow \quad \swarrow & \\ & \Pi_{n,S}S & \end{array}$$

For the simplicity, η denotes a generic point of S . The extension of $\Pi_{n,S}(Y_\eta) \rightarrow \Pi_{n,S}(\eta)$ is to be considered, which has the kernel Poincare-Segal n -groupoid $\Pi_{n,S}(Y_{\bar{\eta}})$ where $\bar{\eta}$ denotes the point associated to the algebraic closure in an algebraically closed field which contains $k(\eta)$. Note that $\Pi_{n,S}(\eta) = \Pi_1(\eta, \bar{\eta})$. Since $\Pi_{n,S}(\eta)$ is the limit of Poincare-Segal n -groupoids $\Pi_{n,S}(U)$ where U runs over all open sets of η , it equals to a usual Poincare group, which is an absolute Galois group. The extension

$$\Pi_{n,S}(Y_{\bar{\eta}}) \rightarrow \Pi_{n,S}(Y_\eta) \rightarrow \Pi_{n,S}(\eta)$$

is determined by the Breen cohomology ([Breen2])

$$H^1(\Pi_1(\eta, \bar{\eta}), (\Pi_{n,S}(Y_{\bar{\eta}}) \rightarrow \text{Aut}(\Pi_{n,S}(Y_{\bar{\eta}}))))$$

On the other hand $\Pi_{n,S}(X)$ admits a section over $\Pi_{n,S}(S)$, so does $\Pi_{n,S}(Y)$. Thus one can consider the cohomology

$$H^1(\Pi_1(\eta), \text{Aut}(\Pi_{n,S}(Y_{\bar{\eta}})))$$

By hypothesis the automorphism group of $Y_{\bar{\eta}}$ is isomorphic to a complex semi-simple group, which is algebraic. Note that

$$\text{Aut}(\Pi_{n,S}(Y_{\bar{\eta}})) \cong \text{Aut}_{\bar{k}}(Y_{\bar{\eta}})$$

Hence replacing S by S' where S'/S is generically finite morphism, the element associated to the section is trivial in the cohomology

$$H^1(\Pi_1(\eta), \text{Aut}(\Pi_{n,S}(Y_{\bar{\eta}})))$$

The canonical homomorphism

$$H^1(\Pi_1(\eta), \text{Aut}(\Pi_{n,S}(Y_{\bar{\eta}}))) \rightarrow H^1(\Pi_1(\eta, \bar{\eta}), (\Pi_{n,S}(Y_{\bar{\eta}}) \rightarrow \text{Aut}(\Pi_{n,S}(Y_{\bar{\eta}}))))$$

implies the triviality of the extension questioned above. One obtains the proof by virtue of the theory of Poincare-Segal n -groupoids. \square

It is a fact that the automorphism group of a projective flat scheme over a locally noetherian scheme is a scheme by Grothendieck using Hilbert scheme method. Hence one has the following theorem in the same argument above:

Theorem 3.9. *Let S be a locally noetherian scheme. Let $f : X \rightarrow Y$ be a surjective morphism between projective flat schemes over S . Suppose X is isomorphic to a product $X_0 \times S$ where X_0 is a projective scheme. Then Y is also a product over a scheme S' where $S' \rightarrow S$ is a generically finite morphism.*

We refer to non commutative algebras and geometries ([AC],[Cohn],[Brz],[Kon],[MO]).

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