

Embodied mathematics and its role in educational playground

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Abstract : Since Lakoff and Nunez's book *Where mathematics comes from: How the embodied mind brings mathematics into being* (2000) was published, there have been many responses from mathematicians and mathematics education researchers. This article includes new ideas and impacts in thinking about mathematics education that arose from their book.

Key words : metaphor, embodied mathematics, mathematical idea analysis

Since Lakoff and Nunez's book *Where mathematics comes from: How the embodied mind brings mathematics into being* (2000) was published, the subject of their book soon became controversial among mathematicians and mathematics education community. The book covered a wide range of subjects and the argument the authors developed in the book was based on many basic evidences from empirical sciences. It was not the first occasion for mathematicians to think about the learning process of mathematics. Many prominent authors, including Descartes, Boole, Dedekind, Poincaré, Cantor, and Weyl, had tried to explain the process of findings in mathematics using the method called self-introspection. Lakoff and Nunez says in the book (Preface, p.XIII) that *as valuable a method as this can be, it can at best tell a partial and not fully accurate story.*

Also, from a mathematician's viewpoint, the book seems to contain mathematics that is rather clumsy. The two authors of the book are not mathematicians; one is a linguist and the other is a cognitive psychologist. Yet, it seems to contain a lot of mathematics—mathematical expositions of undergraduate level. Mathematicians have a style of reading a mathematics book, i.e. they read it backward from the back cover. They search for the main theorem—that which is proven mainly in the book—then go back to prerequisites needed to understand or prove that theo-

rem. If they read Lakoff and Nunez's book in that manner, they would feel uncomfortable, because, mathematically, nothing new is proven in the book and the style of presenting materials is somewhat different from an ordinary mathematics book. What is needed here is a new way of looking at mathematics. The so-called 'papers' produced by professional mathematicians is not the only kind of mathematics we human beings employ. This means that there are several kinds of mathematics, and that we have to make distinction between them if we want to avoid useless confusion.

1 CM and IM—which mathematics are we talking about?

Martin Schiralli and Nathalie Sinclair makes distinction between CM and IM (2003). CM, or *conceptual mathematics*, is the mathematics mainly treated in Lakoff and Nunez (2000). This is mathematics as a subject matter or discipline. Mathematics as a discipline is a public activity. It is an ongoing process of game played by the participants of our society, and the rule of that game should be continuously negotiated and shared. The shared rule of the game constitutes the meaning of mathematics.

A mathematical concept is therefore a publicly accessible tool to manipulate mathematical patterns. That means that CM is outside each in-

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dividual: these CM concepts are not necessarily the same as the mathematical ideas that individual mathematicians (experienced or novice) may form inside of them. How an individual represents these concepts to herself is what is called *ideational mathematics* (IM).

For example, the way how the concept of *derivative* of a function should be understood is represented in a mathematics textbook. CM is a systematic structure of mathematical concepts, which is shared (although not completely shared) by the public. It is a formally well-structured mathematics and can be learned commonly by all the members of the society. Because CM is taught mainly at schools, it is sometimes called the ‘school mathematics’.

On the other hand, the experience of handling IM is a completely personal experience. Even a professional mathematician uses IM in a very intuitive way. And even the same mathematician can use different kinds of IM’s in the same day. For example, there are several kinds of ways the concept of the derivative can be represented within an individual. A derivative can be the slope of a very small line segment constituting a curve. In the image of a person, a curve consists of lots of infinitesimally small line segments. Or, sometimes, a derivative is the slope of a tangent line to the curve, or it is a limit of some formula. Or, it is the speed of motion we feel in our muscle. Though these concepts contradict with each other, mathematicians use some of these simultaneously and deduce results from them very effectively.

Another example is the value of $0.999\ldots$, the digit ‘9’ coming around infinitely. School teachers know that many people believe that $0.999\ldots$ is less than 1. We observe that even overwhelming majority of adults in our society believe so. It should be shocking for a mathematician to know that many of the school teachers of mathematics also believe firmly that $0.999\ldots$ is ‘a little bit’ smaller than 1. Those pupils and adults soon learn to answer that $0.999\ldots$ is equal to one if explained by starting

from the relation $0.333\ldots = 1/3$ and multiplying both sides by three. The important thing here is that even after convincing that $0.999\ldots = 1$, that explanation falls apart very soon. Deep in their heart, people know that it is really a little bit less than one. They just learn that school mathematics is slightly different from the reality they experience in the real world. Of course, they know that school mathematics is useful as a basis for science and engineering. But it is true only in that restricted sense. Truth merely as a tool can be different from the real truth in the depth of the human heart.

The lesson here is that there is not the only one true mathematics. CM is not the only mathematics that is true. It is the common standard of our society, and even CM can change in the course of history.

2 The abstract nature of mathematical thinking

Lakoff and Nunez (2000) argues that *(the) intellectual content of mathematics lies in its ideas, not in the symbols themselves*. They claim that abstract concepts are always rooted, through some combination of linking and grounding metaphors, to sensory-motor experience. They claim that we cannot think of the derivative, for example, without conceptualising it in terms of something more concrete.

But the abstract nature of mathematical thinking goes beyond the concrete ideas.

A mathematician’s concept of the derivative is ‘detached’ from those particular meanings, and, as such, they can be applied to totally different situations.

If we try to think about a triangle, we have a vague image of a triangle in our mind. But when we think of our triangle in our image precisely, then we find that that triangle we are looking at is a particular triangle, not an abstract triangle. We can never make an abstract image of a triangle in our mind, yet a mathematician can treat an abstract triangle and argue about it. How can he do that?

Imagine that someone tells you to think about any integer in your mind and multiply it with 9. What you really do is to take an example of an integer, say 1124. Then the product of it with nine is 10116. We cannot think of the nine times an integer without deciding which integer we are thinking about. But a mathematician can. How? A mathematician can give a name ' n ', for example, to the abstract integer we are thinking at present. Then the multiple of that integer with nine is $9n$. A mathematician can thus think of an arbitrary object, because he has a systematic method of giving names to arbitrary objects. And the skillful 'names' of abstract objects are called the 'letters' or 'variables' or 'symbols'.

The idea is the same as the use of language. A mathematician can think of an abstract triangle, because they skillfully make use of a word like 'triangle'. Then, they can investigate that triangle using logic. They treat abstract objects, solve problems concerning those abstract objects, but thinking linguistically.

Each person has a different image or concrete example to help thinking about an abstract triangle or an integer. When we put it linguistically, an abstract object gets a form and we can transfer that form to other people. That 'form' is the concept or a word representing the object.

The distinction between CM and IM plays a major role in this context.

3 Mathematician's opinion

Certain mathematicians found some of the metaphors used by Lakoff and Nunez (2000) either foreign or forced. One mathematician in logic once said to me that mathematics varies from mathematician to mathematician. This opinion contradicts what is written in the book: *the metaphors on which mathematics is based are not at all arbitrary. That is, grounding metaphors are forced on us by our physical nature, and metaphorical mappings, blends and special cases have a stable, precise structure* (see p.375).

Not only mathematicians, but also learners of mathematics have a variety of metaphors, as every teacher knows. Some of them are preferable and some are not, from the teacher's viewpoint. These metaphors are mentioned *extraneous* metaphors and never treated again in the book.

The reality is that the individual person develops IM in herself. CM is the common way or the mainstreet concept of mathematics, while there are many other conceptions of mathematics in each mathematician and learner.

4 Conclusion

Lakoff and Nunez's concept of embodied mathematics sheds a new light in the philosophy of mathematics. Constructing CM in terms of metaphoric mappings will make a useful standard model for the teacher to understand the structure of learner's mathematics. We should also take into account the fact that the individual learner may have other metaphors to help understand mathematics and construct her own IM.

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