

Group Theoretic Geometry

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1 Introduction:

In this note we shall propose a construction of a new geometry called group theoretic geometry. Possibly it might be a variation of a representation theory of groups. We grasp the concept of a variety in this sense as follows: Given a field F , we patch subrings (R_i) of the field F with the same total quotient field $Q(R_i) = F$ in such a way that there exists a topological space $X = \cup_i U_i$ such that $U_i = \text{Spec} R_i$. Here the subrings (R_i) form an inductive system and a generic point is represented by $F = \varinjlim R_i$. For an affine variety R_i and an irreducible divisor $V(f)$, the generic point of $V(\vec{f})$ is defined by the total quotient of $R_i/(f)$.

Let Γ be a profinite group and $\{H_i\}$ a family of profinite normal closed subgroups of Γ . We patch quotient groups Γ/H_i such that there exists a topological space $X = \cup_i U_i$, where $\pi_1(U_i) = \Gamma/H_i$. The Γ/H_i form a projective system and the limit is $\Gamma = \varprojlim \Gamma/H_i$. For an affine variety $U_i = \text{Spec} R_i$ and a divisor D of U_i , $\pi_1(U_i)$ is a factor group of a profinite group Γ_D of a generic point of D . We have $\Gamma_D = \varprojlim_{g \neq f} \pi_1(R_i[f^{-1}]/(g))$, where $V(g) = D$.

2 Affine scheme

Lemma 2.1. *Let A_p be a local ring and p a prime ideal. Let \hat{A}_p be a completion of A_p . Then $\hat{A}_p \rightarrow A_p$ is faithfully flat and etale. Hence the homomorphism $\pi_1(\text{Spec } \hat{A}_p) \rightarrow \pi_1(\text{Spec } A_p)$ is injective.*

Proof. Since $\hat{A}_p/p \cong A_p/p$, $\text{Spec } \hat{A}_p \rightarrow \text{Spec } A_p$ is unramified and faithfully flat. Hence it is an etale covering. □

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Theorem 2.2. ([21]) *Let $Y = \text{Spec } A$, A a complete noetherian local normal ring with k a residue field, X a proper Y -scheme, $X_0 = A \otimes_A k$, a_0 a geometric point and a the corresponding point of X . Then the canonical homomorphism $\pi_1(X_0, a_0) \rightarrow \pi_1(X, a)$ is an isomorphism.*

Corollary 2.3. *Since \hat{A}_p is a complete noetherian local ring, one has $\pi_1(\text{Spec } A_p/p) \cong \pi_1(\text{Spec } \hat{A}_p)$*

Proof. $\hat{A}_p/p \cong A_p/p$. □

Let k be a field and A a k -algebra of finite type. Let $X = \text{Spec } A$. Assume $A \subset Q(A)$.

Let Γ be the absolute Galois group $\pi_1(Q(A), \overline{Q(A)})$. Let $f \in A$. Then $\pi_1(A[f^{-1}])$ is a quotient Γ/N_f of Γ .

Proposition 2.4. *Let S be an irreducible locally noetherian normal scheme with K the function field. Let Ω be an algebraically closed extension of K . Let a' be a geometric point of $\text{Spec } K$ and a a geometric point of S . Then the homomorphism*

$$\pi_1(\text{Spec } K, a') \longrightarrow \pi_1(S, a)$$

is surjective. The kernel corresponds to the subextension \bar{K}/K composed by finite extensions in Ω unramified over S .

Proof. The maximal etale covering over S induces an unramified extension of K . Let N be the corresponding normal subgroup. One has an isomorphism $\pi_1(S, a) \cong \Gamma/N$. □

Corollary 2.5. *Let $U \subset X$ be an open immersion. Then the homomorphism $\pi_1(U) \rightarrow \pi_1(X)$ is surjective.*

Proposition 2.6. *Let $f : Y \rightarrow X$ be an etale covering and b , a geometric points which corresponds by f over Y , X . Then the homomorphism $\pi_1(Y, b) \rightarrow \pi_1(X, a)$ is injective.*

Definition 2.7. ([21]) *A morphism $f : X \rightarrow S$ is said to be separable if every fibre is reduced.*

Proposition 2.8. *Let $f : X \rightarrow S$ be a proper separable morphism between locally noetherian schemes. Then $\text{Spec } f_*\mathcal{O}_X \rightarrow S$ is an etale covering.*

Theorem 2.9. ([21]) *Let $f : X \rightarrow Y$ be a proper separable morphism with Y locally noetherian connected scheme. Suppose $f_*\mathcal{O}_X = \mathcal{O}_Y$. Let X' be an etale covering of X . There exists an etale covering Y' such that $X' \cong X \times_Y Y'$, if and only if \overline{X}_y admits a section over \overline{X}_y .*

Corollary 2.10. ([21]) *Let \bar{a} be a geometric point of $\overline{X_y}$, a its image in X and b its image in Y . Then the the following sequence is exact:*

$$\pi_1(\overline{X_y}, \bar{a}) \rightarrow \pi_1(X, a) \rightarrow \pi_1(Y, b) \rightarrow 1$$

Corollary 2.11. ([21]) *Let k be an algebraically closed field, X and Y connected schemes over k . Suppose that X is proper over k and Y is locally noetherian. Let a be a geometric point of X , b a geometric point of Y with values in the same algebraically closed extension K of k . Let $c = (a, b)$. The homomorphism $\pi_1(X \times_k Y, c) \rightarrow \pi_1(X, a) \times \pi_1(Y, b)$ is induced by the homomorphisms between the fundamental groups associated to projections $X \times_k Y \rightarrow X$ and $X \times_k Y \rightarrow Y$. This homomorphism is an isomorphism.*

Proposition 2.12. ([21]) *Let $f : X \rightarrow Y$ be a proper separable morphism with Y locally noetherian connected scheme. Suppose $f_*\mathcal{O}_X = \mathcal{O}_Y$. The homomorphism $\pi_1(X) \rightarrow \pi_1(\text{Spec } f_*\mathcal{O}_X)$ is surjective and $\pi_1(\text{Spec } f_*\mathcal{O}_X) \rightarrow \pi_1(Y)$ is injective.*

Let $X = \text{Spec } A$ be a locally noetherian normal irreducible affine scheme and Γ a profinite group $\pi_1(K)$. Let $\pi_1(A[f^{-1}]) = \Gamma/N_f$, where N_f is a normal closed subgroup in Γ . One has $\pi_1(A_p) = \pi_1(\lim_{\rightarrow f \notin p} A[f^{-1}]) = \lim_{\leftarrow f \notin p} \pi_1(A[f^{-1}]) = \lim_{\leftarrow f \notin p} \Gamma/N_f$. So $\pi_1(A_p) = \Gamma / \cap_{f \notin p} N_f$. Let N_p denote $\cap_{f \notin p} N_f$. Next one has

$$\pi_1(A_p/p) \hookrightarrow \Gamma/N_p.$$

Hence there exists a subgroup Γ_p of Γ such that $\pi_1(A_p/p) \cong \Gamma_p/N_p$. Let $\Gamma(p)$ denote Γ_p/N_p . Let $p \subset q$ be prime ideals. Since $A_q \subset A_p$, $\pi_1(A_p) \rightarrow \pi_1(A_q)$ is surjective. Hence $N_p \subset N_q$. Characterizing N_p , one defines a set $X = \{N_p\}$. One gives the closure of a point N_p , $V(N_p) = \{N_q | N_p \subset N_q\}$. Let M be a normal closed subgroup of Γ such that $N \subset M$. Let $V(M) = \{N_p | M \subset N_p\}$. One has $\Gamma/M \cong \pi_1(\lim_{\rightarrow f \in S} A_f)$. Let a be an ideal of A . For $V(a)$ take $\Delta = \cap \{\Gamma_p | \Gamma_p \text{ associate to } p \in V(a)\}$. One has $\Delta/\Delta \cap N \cong \pi_1(A/a)$.

3 Proper Smooth morphisms

Theorem 3.1. ([21]) *Let $f : X \rightarrow Y$ be a proper smooth morphism. Suppose $f_*\mathcal{O}_X = \mathcal{O}_Y$ and Y is locally noetherian. Assume that y_0 and y_1 are points of Y such that $y_0 \in \overline{y_1}$ and that $\overline{X_0}$ and $\overline{X_1}$ are geometric fibres. Then the specialization homomorphism $\pi_1(\overline{X_1}) \rightarrow \pi_1(\overline{X_0})$ is an isomorphism if $k(y_0)$ is of characteristic 0 (resp. $\pi_1(\overline{X_1})^{(p)} \rightarrow \pi_1(\overline{X_0})^{(p)}$ if $k(y_0)$ is of characteristic p).*

Proposition 3.2. *Let $f : X \rightarrow S$ be a proper smooth surjective morphism between complex varieties. Suppose $f_*\mathcal{O}_X \cong \mathcal{O}_S$. Let $g : T \rightarrow S$ be a morphism of finite type. Then $\pi_1(X \times_S T) \cong \pi_1(X) \times_{\pi_1(S)} \pi_1(T)$.*

Proof. By base change $f_T : X_T \rightarrow T$ is a proper smooth morphism. Hence $\pi_1(X \times_S T) \cong \pi_1(X) \times_{\pi_1(S)} \pi_1(T)$. □

Theorem 3.3. *Let $f : X \rightarrow S$ be a proper smooth surjective morphism between complex non singular varieties. Suppose that $f_*\mathcal{O}_X \cong \mathcal{O}_S$ and that $\pi_1(S, a) = 1$. Then $f : X \rightarrow S$ is trivial.*

Proof. One denotes the rational function field of S by $R(S)$. Put $T = \text{Spec}R(S)$. Since $f_T : X_T \rightarrow T$ is a proper smooth morphism, $\pi_1(X_T) \cong \pi_1(X) \times \pi_1(T)$. We will show a proof in the following several steps. □

Lemma 3.4. *Let X be a non singular variety and U, V open varieties. Assume $\text{codim}(U \cup V, X) \geq 2$. Then the following commutative square is cartesian:*

$$\begin{array}{ccc} \pi_1(U \cap V) & \rightarrow & \pi_1(U) \\ \downarrow & & \downarrow \\ \pi_1(V) & \rightarrow & \pi_1(X) \end{array}$$

Proof. Let Γ be the absolute Galois group of the function field $R(X)$ of X and N_X, N_U, N_V normal closed subgroups of Γ which correspond to varieties X, U, V , respectively. Since $N_U/N_U \cap N_V \cong N_U N_V/N_V$, one has the map of the following exact sequences the lower square of which is cartesian:

$$\begin{array}{ccc} 1 & & 1 \\ \downarrow & & \downarrow \\ N_U/N_U \cap N_V & \cong & N_U N_V/N_V \\ \downarrow & & \downarrow \\ \Gamma/N_U \cap N_V & \rightarrow & \Gamma/N_V \\ \downarrow & & \downarrow \\ \Gamma/N_U & \rightarrow & \Gamma/N_U N_V \\ \downarrow & & \downarrow \\ 1 & & 1 \end{array}$$

□

Corollary 3.5. *Let $f : X \rightarrow S$ be a surjective separable connected morphism between non singular varieties. Let U be an open subvariety of X . Assume that $f(U) = S$. Let η be the generic point of S and X_η, U_η generic fibres of $X/S, U/S$, respectively. Then the following commutative square is cartesian:*

$$\begin{array}{ccc} \pi_1(U_\eta) & \rightarrow & \pi_1(U) \\ \downarrow & & \downarrow \\ \pi_1(X_\eta) & \rightarrow & \pi_1(X) \end{array}$$

Proof. Let Γ be the absolute Galois group of the function field $R(X)$ of X and N_X, N_U, N_{X_η} normal closed subgroups of Γ which correspond to varieties X, U, X_η , respectively. Let W be an open subvariety of S . One has $N_U/N_U \cap N_{X_W} \cong N_U N_{X_W}/N_{X_W}$. Since $\lim W = \eta$, one has $\varprojlim_w N_U/N_U \cap N_{X_W} \cong \varprojlim_w N_U N_{X_W}/N_{X_W}$. Since $N_U/N_U \cap N_{X_\eta} \cong N_U N_{X_\eta}/N_{X_\eta}$, one has the map of the following exact sequences the lower square of which is cartesian:

$$\begin{array}{ccc}
 1 & & 1 \\
 \downarrow & & \downarrow \\
 N_U/N_{U_\eta} & \cong & N_X/N_{X_\eta} \\
 \downarrow & & \downarrow \\
 \Gamma/N_{U_\eta} & \rightarrow & \Gamma/N_{X_\eta} \\
 \downarrow & & \downarrow \\
 \Gamma/N_U & \rightarrow & \Gamma/N_X \\
 \downarrow & & \downarrow \\
 1 & & 1
 \end{array}$$

□

Lemma 3.6. *Let $f : X \rightarrow S$ be a proper smooth connected surjective morphism between varieties. Then one obtains the following commutative diagram:*

$$\begin{array}{ccc}
 1 & & 1 \\
 \downarrow & & \downarrow \\
 \pi_1(\overline{U}_\eta) & \cong & \pi_1(\overline{X}_\eta) \\
 \downarrow & & \downarrow \\
 \pi_1(U_\eta) & \rightarrow & \pi_1(X) \\
 \downarrow & & \downarrow \\
 \pi_1(\eta) & \rightarrow & \pi_1(S) \\
 \downarrow & & \downarrow \\
 1 & & 1
 \end{array}$$

Proof. One has the following commutative diagram:

$$\begin{array}{ccc}
 1 & & 1 \\
 \downarrow & & \downarrow \\
 \pi_1(\overline{X}_\eta) & \cong & \pi_1(\overline{X}_\eta) \\
 \downarrow & & \downarrow \\
 \pi_1(X_\eta) & \rightarrow & \pi_1(X) \\
 \downarrow & & \downarrow \\
 \pi_1(\eta) & \rightarrow & \pi_1(S) \\
 \downarrow & & \downarrow \\
 1 & & 1
 \end{array}$$

Thus one interprets the commutative diagram above in the following one:

$$\begin{array}{ccc}
 1 & & 1 \\
 \downarrow & & \downarrow \\
 M/N_X & \cong & N/N_{X_\eta} \\
 \downarrow & & \downarrow \\
 \Gamma/N_X & \rightarrow & \Gamma/N_{X_\eta} \\
 \downarrow & & \downarrow \\
 \Gamma/M & \rightarrow & \Gamma/N \\
 \downarrow & & \downarrow \\
 1 & & 1
 \end{array}$$

Hence one has $M/N_X \cong N/N_{X_\eta}$.

This implies $M/N_U \cong N/N_{U_\eta}$.

Consider the following commutative diagram:

$$\begin{array}{ccc}
 1 & & 1 \\
 \downarrow & & \downarrow \\
 N_{X_\eta}/N_{U_\eta} & \cong & N_X/N_U \\
 \downarrow & & \downarrow \\
 N/N_{U_\eta} & \rightarrow & M/N_U \\
 \downarrow & & \downarrow \\
 N/N_{X_\eta} & \cong & M/N_X \\
 \downarrow & & \downarrow \\
 1 & & 1
 \end{array}$$

Since the lower and upper horizontal arrows are isomorphic, the middle horizontal arrow is an isomorphism by the precedent lemma. □

Lemma 3.7. *Let $g : T \rightarrow S$ be an arbitrary morphism and U an arbitrary open subvariety of X . Assume $\pi_1(U_T) = \pi_1(\overline{U}_\eta) \times \pi_1(T)$. Then there exists a variety F such that $X \cong F \times S$ as S -scheme.*

Hence we complete a proof of the theorem above.

We use the tools of L.Breen([8],[9],[10]) in the proof of the following theorem.

Theorem 3.8. *Let $f : X \rightarrow S$ be a proper smooth surjective morphism between non singular complex varieties. Assume*

- (a) $f_*\mathcal{O}_X = \mathcal{O}_S$
- (b) the generic general fibre of $f : X \rightarrow S$ is of general type.
- (c) $S = S_1 \times S_2$, where S_1, S_2 are varieties.
- (d) for any point $b \in S_2$ the restriction $f_{1b} : X_{1b} \rightarrow S_1 \times \{b\}$ of $f : X \rightarrow S$ has no fixed part over S_1 .

Then there exists S_3 such that for any point $a \in S_1$ the base change $f_{2a,S_3} : X_{2a,S_3} \rightarrow \{a\} \times S_3$ of the restriction of $f_{2a} : X_{2a} \rightarrow S_2 \times \{a\}$ is trivial.

Proof. We will give an outline of a proof in several steps in the following lemmas and propositions. □

Proposition 3.9. *Let $1 \rightarrow G \rightarrow E \rightarrow P \rightarrow 1$ be an extension of profinite groups. There exist the following commutative diagram and exact sequence:*

$$\begin{array}{ccccccccc}
 1 & \rightarrow & G & \rightarrow & E & \rightarrow & P & \rightarrow & 1 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 1 & \rightarrow & \text{Inn}G & \rightarrow & \text{Aut}G & \rightarrow & \text{Out}G & \rightarrow & 1
 \end{array}$$

and

$$0 \rightarrow H^1(P, ZG) \rightarrow H^0(P, (G \rightarrow \text{Aut}G)) \rightarrow H^0(P, \text{Out}G) \rightarrow$$

$$H^2(P, ZG) \rightarrow H^1(P, (G \rightarrow \text{Aut}G)) \rightarrow H^1(P, \text{Out}G)$$

Lemma 3.10. *Let $1 \rightarrow G \rightarrow E \rightarrow P \rightarrow 1$ be an extension of profinite groups. Let $Q \triangleleft P$ be a normal closed group of G . The following sequence is exact:*

$$1 \rightarrow H^1(P/Q, ZG^Q) \rightarrow H^1(P, ZG) \rightarrow H^1(Q, ZG)^{P/Q}$$

Proof. It is the Edge sequence of Serre Spectral sequence([31]). □

In particular, one has the following lemma.

Lemma 3.11. *Let $P = P_1 \times P_2$. The following sequence is exact: for $\{i, j\} = \{1, 2\}$*

$$0 \rightarrow H^1(P_j, ZG^{P_i}) \rightarrow H^1(P, ZG) \rightarrow H^1(P_i, ZG)^{P_j}$$

Lemma 3.12. *For $\xi \in H^1(P, (G \rightarrow \text{Aut}G))$, let ξ_i be the image element of the following composite homomorphism*

$$H^1(P, (G \rightarrow \text{Aut}G)) \xrightarrow{\text{res}} H^1(P_i, (G \rightarrow \text{Aut}G)) \rightarrow H^1(P_i, \text{Out}G)$$

Then there exists a profinite subgroup P_i^ of P_i such that ξ_i is a distinguished element of $H^1(P_i^*, \text{Out}G)$. Hence there exists $\zeta_i \in H^2(P_i^*, ZG)$ such that the image of ζ_i is $\text{res}(\xi)$ by $H^2(P_i^*, ZG) \rightarrow H^1(P_i^*, (G \rightarrow \text{Aut}G))$.*

Proof. Since $\text{Out}G$ is a finite group, it is possible to put $P_i^* = \ker(\xi_i : P_i \rightarrow \text{Out}G)$. □

Lemma 3.13. *One can take a profinite subgroup P_i^{**} of P_i^* such that $H^1(P_i^{**}, ZG) = 0$ for $i = 1, 2$. Denoting by $P^{**} = P_1^{**} \times P_2^{**}$. One has $H^1(P^{**}, ZG) = 0$.*

Proof. Take $P_i^{**} = \bigcap \ker(P_i^* \rightarrow ZG) \supset [P_i^*, P_i^*] \neq 1$. Then $\text{Hom}(P^{**}, ZG) = \prod_{i \in \{1, 2\}} \text{Hom}(P_i^{**}, ZG) = 1$. □

Lemma 3.14. *([31]) Let $Q \triangleleft P$ be a normal profinite subgroup of P . If $H^1(P, ZG) = 0$, then there exists an exact sequence:*

$$0 \rightarrow H^2(P/Q, ZG^Q) \rightarrow H^2(P, ZG) \rightarrow H^2(Q, ZG)^{P/Q}$$

Proof of theorem. Replace $P^{**}, P_i^{**}, P_j^{**}$ by P, P_i, P_j , respectively. Hence one can assume $H^1(P, ZG) = 0, H^1(P_i, ZG) = 0, H^1(P_j, ZG) = 0$. Let $\xi \in H^1(P, (G \rightarrow \text{Aut}G))$ be an extension $: 1 \rightarrow G \rightarrow E \rightarrow P \rightarrow 1$ as in the theorem. The image of ξ by $H^1(P, (G \rightarrow \text{Aut}G)) \rightarrow H^1(P, \text{Out}G)$ is a distinguished element.

Hence there exists an element $\zeta \in \overline{H}^2(P, ZG)$ such that $\alpha(\zeta) = \xi$ by $\overline{H}^2(P, ZG) \xrightarrow{\alpha} \overline{H}^1(P, (G \rightarrow \text{Aut}G))$. Here the overline of $\overline{H}(P, ZG)$ indicates the reduced cohomology. Consider the following exact sequence:

$$0 \rightarrow \overline{H}^2(P_j, ZG^{P_i}) \xrightarrow{\psi_j} \overline{H}^2(P, ZG) \xrightarrow{\phi_i} \overline{H}^2(P_i, ZG)^{P_j}$$

Take the extension $\xi \in H^1(P, (G \rightarrow \text{Aut}G))$ of P such that $\phi_1(\xi)$ has no fixed part restricting to P_1 . Suppose the image $\zeta - \psi_1\phi_1(\zeta)$ is not a distinguished element in the cohomology $\overline{H}^2(P, ZG)$. Then the image $\zeta - \phi(\psi_1(\phi_1(\zeta)))$ is a distinguished element in the cohomology $\overline{H}^2(P_1, ZG)^{P_2}$. Hence there exists an element η_2 such that $\psi_2(\eta_2) = \zeta - \psi_1\phi_1(\zeta)$. Consider the following exact sequence: $0 \rightarrow \overline{H}^2(P, ZG) \rightarrow \overline{H}^1(P, (G \rightarrow \text{Aut}G)) \rightarrow \overline{H}^1(P, \text{Out}G)$. Thus the extension ξ has a non void fixed part restricting to P_1 , which is a contradiction. □

Therefore we complete a proof of the theorem above.

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