Simple Fabrication Techniques of Fourier Transform Hologram with Gray Level/Quasi-Colors and Visual Appearance of Optically Reconstructed Diffractive Images

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A binary computer generated hologram (CGH) and a Fourier transform hologram (FTH) with gray level/quasi-colors are similar to all appearance in the structure of hologram cell. However, they are wholly distinct each other in display techniques. We directly take a picture of reduced size of FTH displayed on a standard color LCD/CRT using a special negative film with high contrast in this study. A few examples of optical diffractive images are demonstrated for the comparison of the proposed FTH techniques with the classic binary CGH techniques from the viewpoint of visual appearance and image quality.

1. Introduction

An optical and ordinary hologram is produced by recording the interference pattern which is caused by an object wave and a reference wave. On the other hand, an important feature of computer generated hologram (CGH) is to create wavefronts that can be defined only mathematically. The CGH produced by a computer permits the generation of wavefronts with any specified amplitude and phase distribution with non-negative values as outlined in the selected papers on "CGHs and diffractive optics" edited by Sing H. Lee in 1992 [1]. There are two main approaches. The first is to calculate the complex amplitude of the object wave and reference wave at the hologram plane. This is usually taken to be the Fresnel or Fourier transform of the complex amplitude in the object data. This technique is analogous to off-axis optical holography. The second uses only the computed values of the discrete Fourier transform (DFT) to produce a special transparency (i.e., CGH) which reconstructs the object wave when it is optically and suitably illuminated using a He-Ne or a LD laser.

In this study, a simple fabrication technique of Fourier transform hologram (FTH) with gray level/quasi-colors is proposed without using a XY plotter device. The visual appearance of optically reconstructed diffractive images is discussed in contrast to the classic binary computer generated hologram (CGH).

2. Optical Fourier Transform Holography

A technique of optical Fourier transform holography is introduced in a distinguished book [2]. A typical optical setup for recording and reconstructing the FTH is shown in **Figure 1**. The 2 D object, a transparency located in the front focal plane of a convex lens, is illuminated by a parallel

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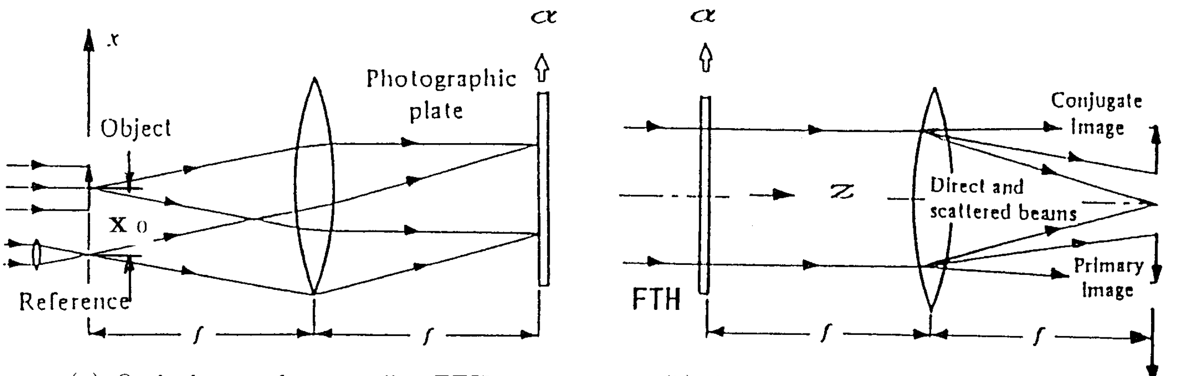

(a) Optical setup for recording FTH (b) Optical setup for reconstructing FTH

Fig. 1 Optical holography setup for Fourier transform hologram (FTH)

beam of monochromatic light such as a laser. If the complex amplitude leaving the object plane is $o(x,y)$, the complex amplitude of the object wave at the photographic plate located in the back focal plane of the lens is described by the following expression [3].

$$O(\alpha, \beta) = F\,[o(x,y)] \tag{1}$$

where, F : Fourier operation symbol

α, β : spatial frequency coordinate

x,y : real spatial coordinate

The reference beam is derived from a point source also located in the front focal plane of the convex lens. If $\delta(x+x_0, y)$ is the complex amplitude of this point source, the complex amplitude of the reference wave corresponding to a tilted plane wave at the hologram plane is described by the following expression.

$$R(\alpha, \beta) = F[\delta(x+x_0, y)] = \exp\{-j(2\pi\alpha x_0)\} \tag{2}$$

where, x_0 : displacement (or shift) of point source against original object <See Fig.1 (a). >

$$\delta(x, y) = \begin{cases} \infty, & x; y=0 \\ 0, & \text{otherwise} \end{cases}$$

The intensity in the interference pattern formed by these two waves is defined as follows.

$$\begin{aligned} I(\alpha, \beta) &= |O(\alpha, \beta) + R(\alpha, \beta)|^2 \\ &= 1 + |O(\alpha, \beta)|^2 \\ &\quad + O(\alpha, \beta)\exp\{+j(2\pi\alpha x_0)\} \\ &\quad + O^*(\alpha, \beta)\exp\{-j(2\pi\alpha x_0)\} \end{aligned} \tag{3}$$

where, $*$: symbol of conjugate complex numbers

j : imaginary unit

To reconstruct the 2 D-image, the photographically processed hologram is placed in the front focal plane of the lens and is illuminated with a special parallel laser beam as shown in Fig.1 (b). If it is assumed that the amplitude transmittance of the processed hologram is a linear function of $I(\alpha, \beta)$, the complex amplitude of the light transmitted by the hologram is formulated by the following expression.

$$H(\alpha,\ \beta)=b+k\times t\times I(\alpha,\ \beta) \tag{4}$$

where, b : constant background transmittance

t : exposure time

k : slope of amplitude transmittance versus exposure characteristic of photographic material

The complex amplitude in the back focal plane of the lens is the inverse Fourier transform of $H(\alpha,\ \beta)$.

$$\begin{aligned} h(x,y) &= F^{-1}[H(\alpha,\ \beta)] \\ &= \{b+k\times t\times\delta(x,y)\}+\{k\times t\}o(x,y)\bigstar o(x,y) \\ &\quad +\{k\times t\}o(x-x_0,y)+\{k\times t\}O^*(-x+x_0,-y) \end{aligned} \tag{5}$$

where, ★ : correlation symbol

F^{-1} : inverse Fourier operation symbol

The first term on the right-hand side of Eq.(5) comes to a focus on the axis, while the second term forms the scattered beam, i.e., a halo around it. The third term corresponds to the original object wave shifted downwards by a distance: x_0, while the fourth term is the conjugate of the original object wave inverted and shifted upwards by the same amount: x_0. Both the images are real and can be observed on a screen or recorded on a photographic film placed in the back focal plane of the lens. Since the film records only the intensity distribution in the image, it is possible to identify the conjugate image.

Note that the phase factor contained in Eq.(3) has no effect on the intensity distribution, but has an effect of a shift of the position of reconstructed image in the case of reconstruction process.

The dot resolution of a standard color LCD/CRT is often composed of 640×400 dots. A simple fabrication of FTH can be performed using 64×64 pixels, on condition that the allowable maximum size of unit hologram cell consists of 6×6 dots per one pixel. The image quality of reconstructed

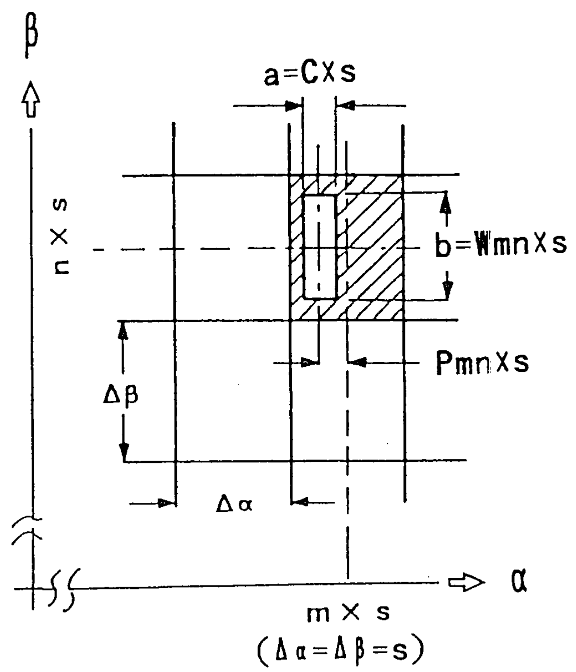

Fig. 2 Typical structure of unit hologram cell for FTH

images is discussed considering the dot number of unit hologram cell, i. e., 5×5 and 5×6 dots.

3. Fabrication of Fourier Transform Hologram and Simulation Techniques

3.1 Aperture function of hologram cell

Figure 2 shows a part of binary hologram cell. Each hologram cell, i.e., transparency with non-negative values, can be formulated. In this study, We use the hologram cell as shown in Fig.2, because of the simple fabrication of FTH similar to the binary CGH. This hologram cell is called a binary "detour-phase hologram". The rectangular opening has a width: C×s, and a height:Wmn× s. It is displaced from the center of the hologram cell at (m×s,n×s) by Pmn×s. An aperture function with the rectangular opening is defined as follows.

$$H(\alpha,\ \beta)=\Sigma\Sigma Gmn\times rect\left[\frac{\alpha-(m+Pmn)s}{C\times s}\right]\times rect\left[\frac{\beta-ns}{Wmn\times s}\right] \qquad (6)$$

where, Gmn : intensityor gray level at position (m,n)

M ; N : total number of data (sampling point) in α and β directions

$\Sigma\Sigma=\sum_{m=0}^{M-1}\sum_{n=0}^{N=1}$: abbreviated symbol

$$rect(x)=\begin{cases}1, & |x|\leqq 1/2\\ 0, & \text{otherwise}\end{cases}$$

3.2 Reconstruction image from FTH

Reconstruction image from binary CGH or general FTH can be formulated using the aperture function and inverse Fourier transform.

$$\begin{aligned}h(x,y)&=rect(x/D)\,rect(y/D)\iint H(\alpha,\ \beta)\exp\{j\,2\,\pi x_0\,\alpha\}\\&\quad\times\exp\{j\,2\,\pi(x\alpha+y\beta)\}d\alpha d\beta\\&=rect(x/D)\,rect(y/D)\times C\cdot s^2\,sinc\{C\cdot s(x_0+x)\}\\&\quad\times\Sigma\Sigma Gmn\times\ [Wmn\times sinc\{yWmn\times s\}\\&\quad\times\exp\ \{j\,2\,\pi s[(x_0+x)\cdot(m+Pmn)+ny]\}]\end{aligned} \qquad (7)$$

where, D : size of image data in real spatial coordinate

s : size of each hologram cell in spatial frequency

(minimum spacing of unit hologram cell)

$sinc(x)=\sin(\pi x)/\pi x$

$\iint=\int_\alpha\int_\beta=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}$: abbreviated symbol

The original Fourier spectrum can be reconstructed by the sampling theorem from the discrete sampling data of Fourier spectrum in the case of the domain of spatial frequency.

$$F(\alpha,\ \beta)=\Sigma\Sigma F(ms,ns)\times sinc(\alpha/s-m)\,sinc(\beta/s-n) \qquad (8)$$

where, $F(ms,ns)=Amn\times\exp\{j\,2\,\pi(\phi_{mn}/2\,\pi)\}$

The original image can be formulated and reconstructed using the following inverse Fourier transform.

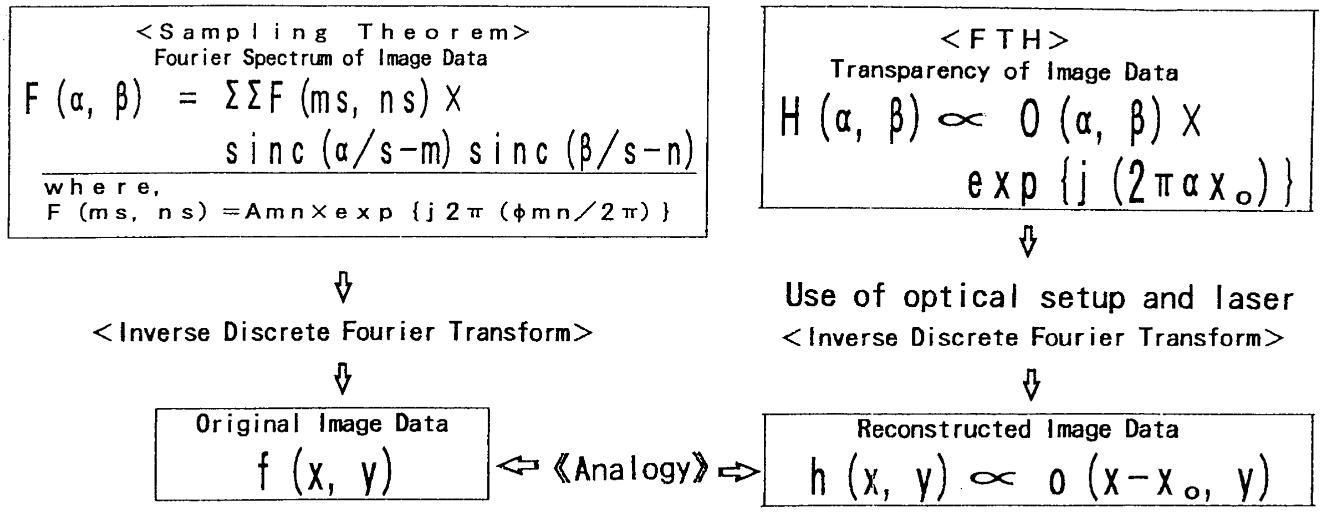

Fig. 3 Analogy between original data by sampling theorem and reconstructed data using FTH

$$f(x,y)=\iint F(\alpha,\ \beta)\exp\{j\,2\,\pi(x\alpha+y\beta)\}d\alpha d\beta$$
$$=\mathrm{rect}(x/D)\,\mathrm{rect}(y/D)\sum\sum[Amn\cdot\exp\{j\,2\,\pi(\phi_{mn}/2\,\pi)\}$$
$$\times\exp\{j\,2\,\pi s(xm+yn\}] \quad (9)$$

Figure 3 shows the relation between original image data and reconstruction data by the inverse Fourier transform techniques.

Reconstruction image from Fourier transform (binary) hologram can be formulated using the aperture function and inverse Fourier transform.

We can compare the each term:$\sum\sum$ in Eqs.(7) and (9), on condition that h(x,y) in Eq.(7) is approximately proportional to f(x,y) in Eq.(9). The following expressions hold as for amplitude and phase term, respectively [5]. It is possible to calculate the complex amplitude: amplitude and phase components in the each hologram cell [4].

$$\text{Amplitude term}: Wmn=Amn/Gmn \quad (10)$$

We can obtain the following expression, i.e., the displacement or shift from the center position in each hologram cell, assuming that a few approximate expressions hold good.

$$\text{Phase term}: Pmn=\phi_{mn}/2\,\pi K \quad (11)$$

where, $K=sx_0$

ϕ_{mn} : shift from center point (sampling point) of each hologram cell

Table 1 shows the hybrid information process for CGH. Note that a plotter is not used. The hybrid process consists of two procedures:<Calculation of FTH> and <Analog (Optical) Information Process>[5]. It is simply possible to calculate the complex amplitude of 2D-image data using DFT.

4. Simple Fabrication of FTH and Optical Diffractive Images

Generally, an optical hologram equivalent to binary CGH/FTH can be made by the optical setup shown in Fig.1(a) and photography techniques. A positive type hologram is simply displayed using monochromatic gray level or quasi-color on a standard color LCD/CRT.

Table 1 Hybrid Information Process for FTH

<Digital Information Process>

(a) Caluculation of FTH:
Amplitude + Phase components with non-negative values using Fresnel or Fourier transform formula

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(b) Digital Image Processing:
Display of FTH on LCD/CRT

<Analog (Optical) Information Process>

(c) Photographing of FTII:
Photographcally reduced transparency

⇩

(d) Optical Image Processing:
Reconstruction of FTH, i.e., monochrome transparency using optical setup and laser

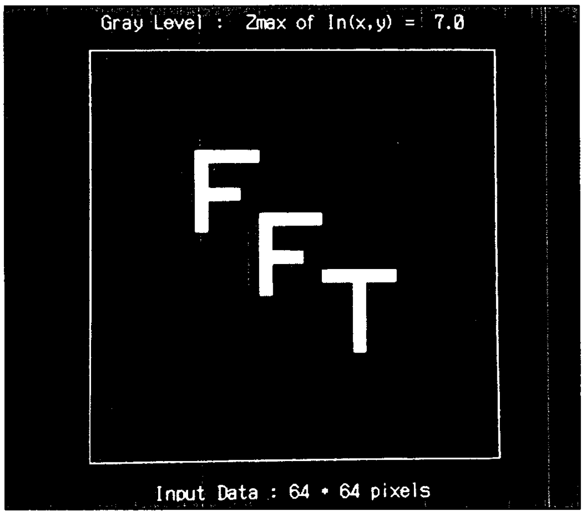
Table 2 Typical Computer Generated Hologram and Its Characteristics

Type of CGH	Characteristics
(a) Interference pattern with monochromatic gray level	Simulation technique similar to optical holography Intensity of object and reference waves $I(\alpha,\beta) = \lvert O(\alpha,\beta) + R(\alpha,\beta) \rvert^2$
(b) Superposition of amplitude and phase components with gray level	Direct CGH ・・・ Double Exposure Simulation technique of complex amplitude transmittance with non-negative values, $T(\alpha,\beta) = Amn \times \exp\{j\phi(\alpha,\beta)\}$
(c) Binary computer generated hologram with aperture	Classic CGH Amplitude: height of aperture, Phase: shift of center position of aperture, Use of fine display device and XY plotter
(d) Fourier transform hologram with gray level or quasi-colors	Simplified CGH ・・・ Single Exposure Application of technique (c), Use of standard LCD/CRT and direct fabrication of transparency

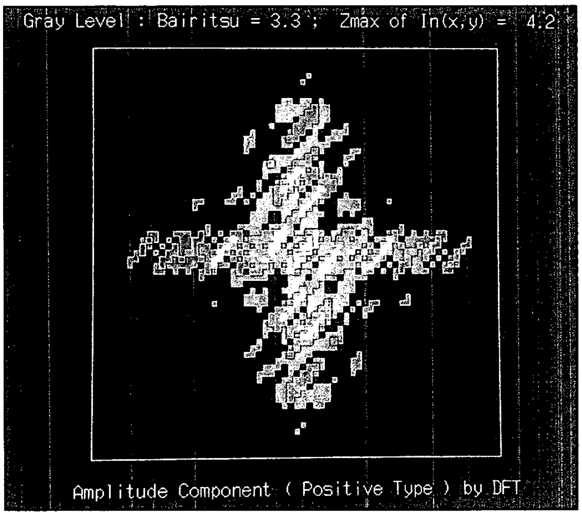
By means of the hard copy (ex. plotter, printer, etc.), the positive type hologram is converted to the negative type hologram. It is possible to fabricate the transparency corresponding to the positive type hologram which is optically reduced in size, when using a negative film with high contrast.

Table 2 shows the typical computer generated hologram and its characteristics. It is possible to calculate the complex amplitude of 2 D-image data using the discrete Fourier transform (DFT). A Fourier transform hologram (FTH) can not be simultaneously displayed by the superposition of amplitude and phase components on a color LCD/CRT. It is necessary to take a photo of double exposure in the case of Table 2 (b).

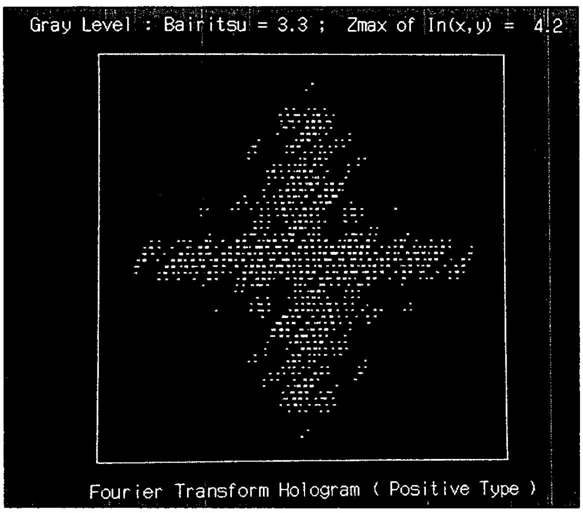
The amplitude component is proportional to the height having no relation to gray level and/or to gray level under the constant height in the hologram cell. On the other hand, the phase component is proportional to the specified displacement from the center of hologram cell in the case of Table 2(c). We directly take a photo of photographically reduced hologram using a negative film with



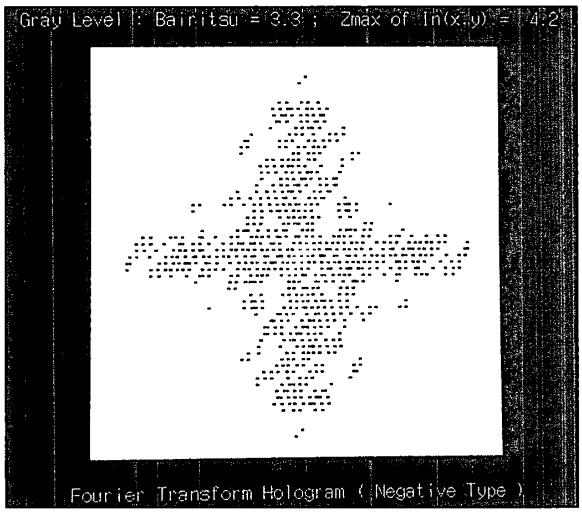
(a) Input data with three characters



(b) Amplitude component by DFT

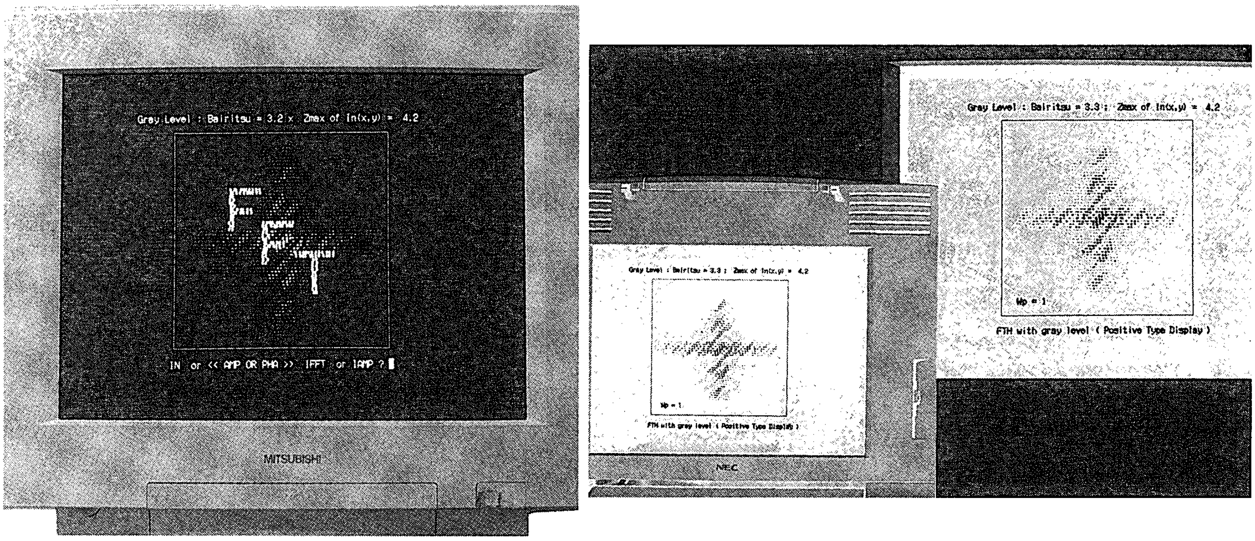


(c) FTH (Positive Type)



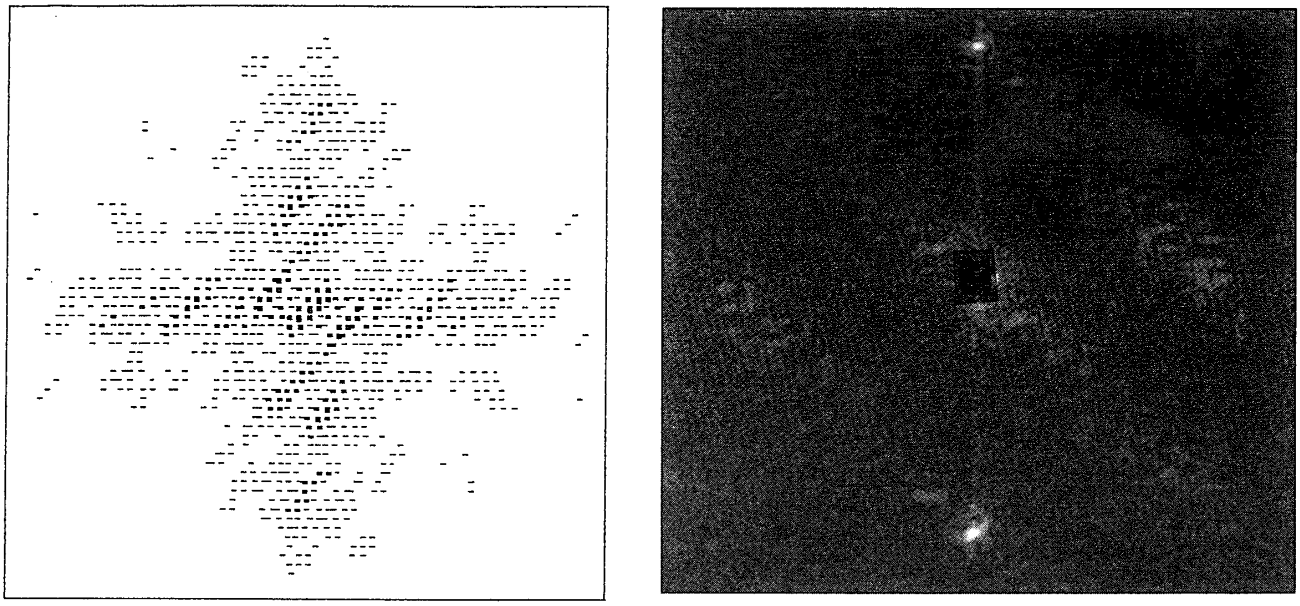
(d) FTH (Negative Type)

Fig. 4 Input data, amplitude component and FTH in quasi-colors in the case of 64×64 pixels



(a) Display containing modified input data on CRT (b) FTH with gray level

Fig. 5 CGH display results



(a) Negative type hologram <Hard copy of Fig. 5 (b)> (b) Diffractive image reconstructed using reduced positive transparency

Fig. 6 Enlarged FTH and reconstructed image

the high contrast characteristic. If the transparency is illuminated with a laser, a reconstructed diffractive image is observed on a translucent screen in the optical setup [5].

Figures 4 (a), (b) and (c) show an input data, an amplitude component, and FTH, respectively. Fig. 4 (a) shows the original image including three simple characters with a uniform gray level. The number of pixel resolution is 64×64. Fig.4 (b) shows the amplitude component by the discrete Fourier transform (DFT). Fig.4(c) demonstrates the FTH (positive type) with the quasi-colors. Though Fig. 4 (c) has the same pattern against Fig.4(b), the former has the phase component. On the other hand, the latter has only the amplitude component.

Figures 5 (a) and (b) show the modified input patterns and the FTH displayed on a standard

CRT. Fig. 5 (b) displayed in monochromatic color shows the same FTH in contrast to Fig. 4 (c) displayed in quasi-colors.

Figures 6 (a) and (b) show an enlarged negative types with bilevel corresponding to Fig.4 (c) or Fig. 5 (b), and the optically reconstructed diffractive image. In Fig. 6 (a), the height of hologram cell: Wmn×s, and the displacement from the center of sampling point: Pmn×s can be calculated from the complex amplitude by the discrete Fourier transform (DFT). The width of hologram cell is fixed as a constant parameter: C×s=(1/2)s.

In Fig. 6 (b), the reconstructed image is demonstrated by the reduced positive type transparency, i.e., the use of negative film with high contrast. The original characters are not clear, and the reconstruction image has a little noises on a translucent screen, because many parameters are used in the simulation process. It is difficult to find a best combination of parameters in advance.

Generally, a combination of appropriate parameters must be examined by trial and error against our will.

Many parameters such as the total number of pixels, the variable height, the shift of the center of sampling point, the number of gray level or quasi-color, etc. are introduced into the simulation technique of FTH. Note that a kind of the negative type FTH must be directly displayed on a color LCD/CRT for fabricating the photographically reduced positive transparency. A commercially available negative film, called minicopy film with high contrast in the H & D characteristic curve, is used in this study.

<Discussions>

It takes only a few minutes on the NEC PC-9821 Nx to fabricate the FTH for a 2 D-image consisting of 64*64 sampling points for DFT. The main characteristic of the proposed FTH is as follows. A computer generated positive type hologram is obtained from a FTH of the negative type on a standard color LCD/CRT by the direct photography techniques (the use of a negative film). On the other hand, a hard copy as the first step, i.e., an enlarged negative type CGH, is plotted from the display result of the positive type on a standard color LCD/CRT in the case of the binary CGH. As the next step, a photograph on a reduced size is taken from the hard copy.

There are a few problems such as a combination of parameters in the simulation process on how to fittingly display the negative type hologram instead of the positive type hologram.

The dynamic range of Fourier spectrum often called transform coefficients is much larger than the exposure range of photographic film. It is necessary to compress the peak value of coefficient to produce a useful display. Generally, compression can be obtained by clipping the large magnitude of peak value or by taking the logarithm of each magnitude value: $|H(\alpha, \beta)|$ according to the following modified relation.

$$Hm(\alpha, \beta)=\log_{10}\{a+b\times|H(\alpha, \beta)|\} \tag{12}$$

where, a;b: scaling constant (ex. a=1 and b=10~15)

Use of this relation preserves the zero values in the frequency plane, since modified FTH:Hm becomes zero when $|H(\alpha, \beta)|=0$ holds good. Many image spectra decrease rather rapidly as a function of frequency. Therefore, their high-frequency terms have a tendency to become surely obscured or suppressive when displayed in image form. The binary CGH and gray level FTH seem to be similar in form. They, however, are wholly distinct each other in display techniques.

A direct picture, i.e., positive or negative transparency of FTH, displayed on a color LCD/CRT is used in this study. As a result, a photographically reduced hologram with a specified gray level or quasi-color can be simply produced. There are a few problems and unique aspect for simple fabrication techniques of FTH as follows.

- The number of total pixel relevant to the size of each hologram cell, i.e., the resolution number of LCD/CRT by increasing or decreasing the number of sampling points can be altered to check the effect of visual appearances and image quality.
- In order to compress the peak value of the Fourier spectrum and to enhance the amplitude of high frequency component, the logarithm presentation of Eq. (12) is introduced.
- Direct photographing techniques using a negative film with the high contrast in place of a standard monochrome or color film are available for a simple fabrication of FTH.
- It is difficult to decide the appropriate values of parameters used in the simulation program for the fabrication of FTH in advance. This necessitates trial and error for us.
- It is impossible to carry out the superposition of two transparencies, i.e., display techniques of negative type FTH on LCD/CRT for amplitude and phase distribution with non-negative values.
- A few examples of optical diffractive images are demonstrated to compare proposed FTH techniques with the classic binary CGH techniques from the viewpoint of visual appearance and image quality. We can not observe the clear diffractive images at present, because it is difficult to find the combination of appropriate parameters, e.g., parameters for clipping the magnitude of peak value, used in the simulation techniques.
- It is possible to discuss the effect owing to the spatial low-pass filter or mask, i.e., the cutoff of complex amplitude with high frequency component in the case of FTH.

<Photography of Fourier transform hologram (FTH)>

It should be noted that photography of Fourier transform hologram (FTH) has an effect on the visual appearance and quality of optically reconstructed diffractive images. Two types of FTHs, i.e., positive and negative type displays are simply demonstrated on a color LCD/CRT. It is necessary for us to consider the following conditions for taking a photo of FTH in advance.

(a) A proper distance between display surface and camera position for fabricating the FTH less than 10 mm × 10 mm in reduced size:

(b) A rational combination of exposure time: T and stop: f for a negative film or color reversal film:

(c) Adjustment of screen lunimance on a standard LCD/CRT in connection with photographic techniques:

5. Conclusion

In this study, the rectangular aperture (or opening) in each hologram cell is not binary (black and white) type. It has the gray level/quasi-colors, because of the simple fabrication and display techniques of FTH. Special display device such as a XY plotter is not necessary. This FTH, however, has more or less noise in the reconstructed image against the binary CGH which is based on diffraction theory. Further discussions should be made from the viewpoint of dot resolution and photographic techniques as well as the combination of appropriate parameters in the simulation algorithm.

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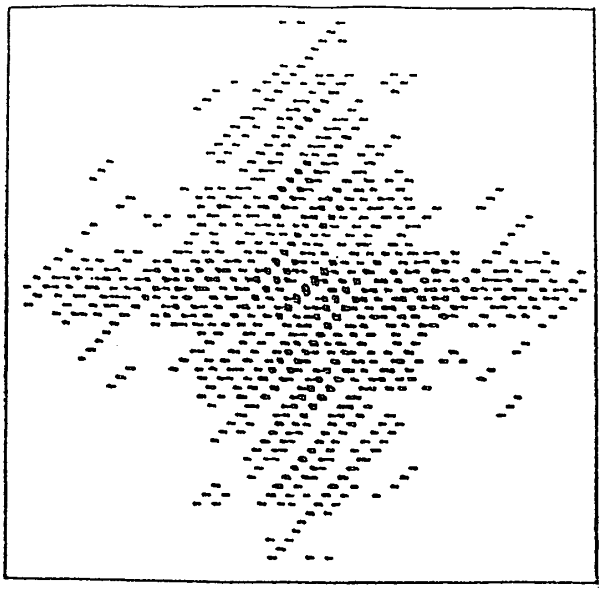
<Appendix>

Typical binary CGHs obtained by a plotter, i.e., photographically reduced holograms with black and white values only, together with a few examples of optical diffractive images, are demonstrated to compare our simple FTH techniques with the classic binary CGH techniques from the viewpoint of image quality.

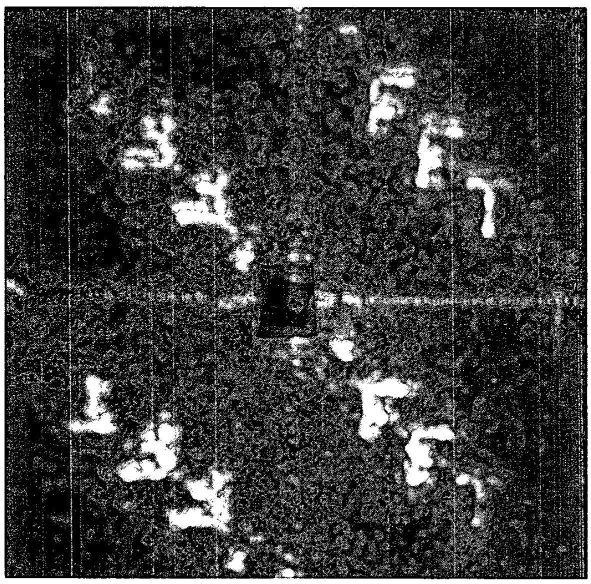
It is possible to discuss the effect of the spatial low-pass filter or mask, i.e., that of the cutoff of complex amplitude with high frequency component.

Figures A (1) and (2) show the binary CGH with three characters composed of 64×64 pixels, and the optically reconstructed diffractive image. The reconstructed image is very clear against in Fig. 6 (b).

Figures B (1) and (2) as well as Fig.A demonstrate the optically reconstructed diffractive image

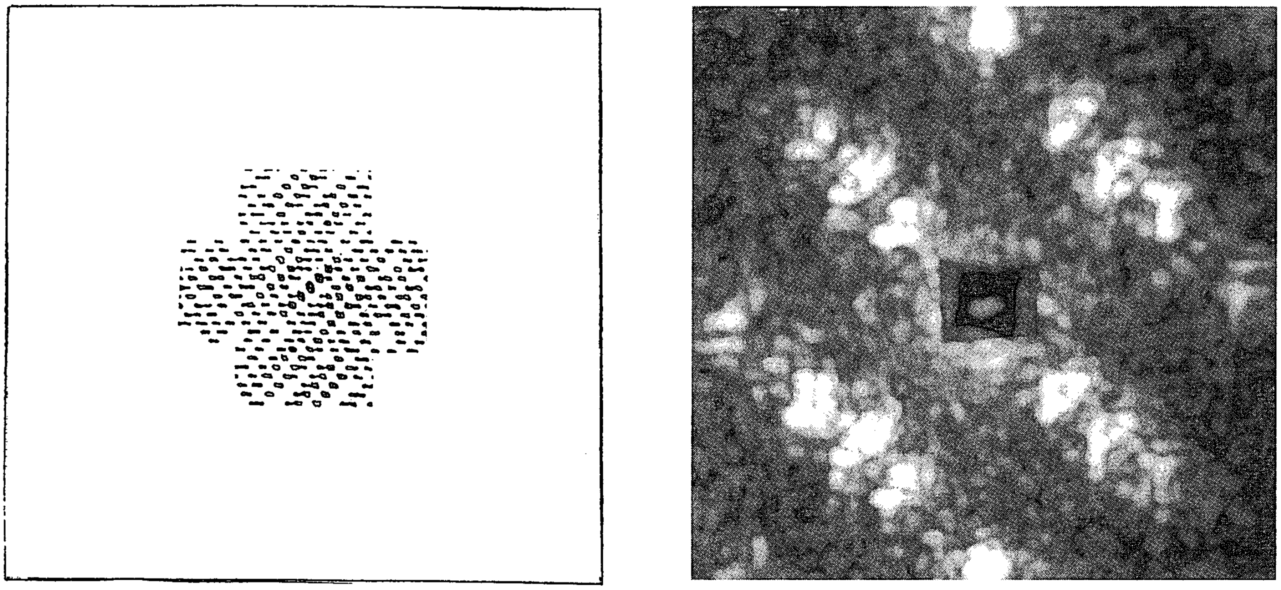


(1) Negative type hologram 〈Hard copy〉

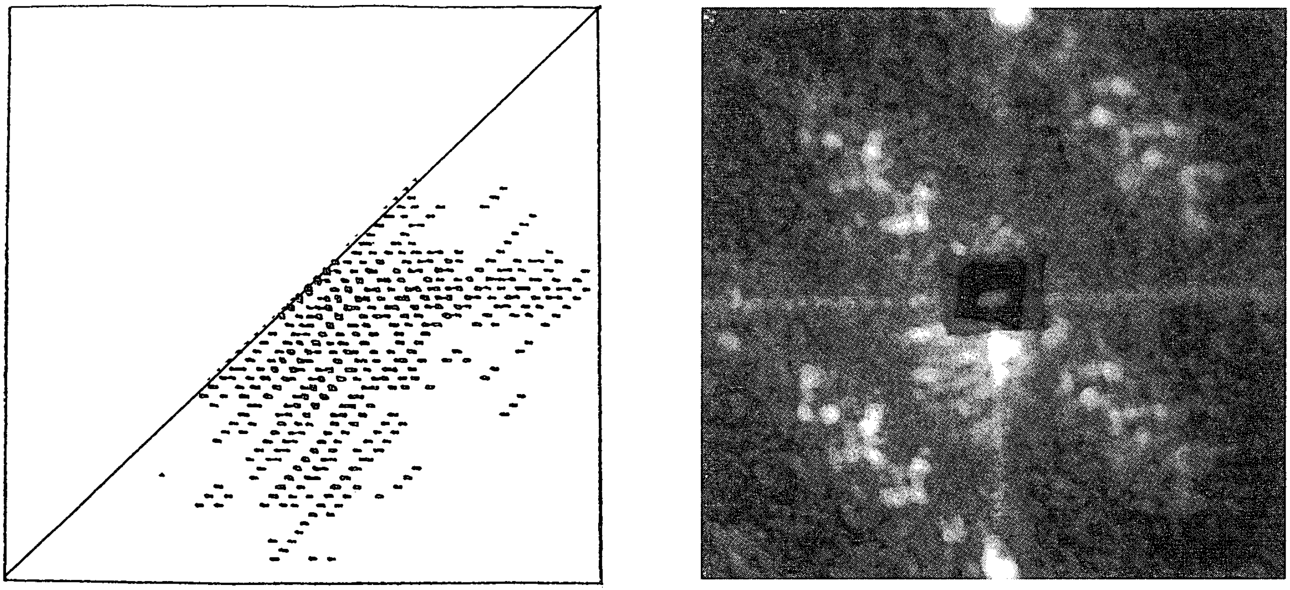


(2) Use of reduced positive transparency

Fig. A Enlarged binary CGH and reconstructed image using a part of Fig.A (1), i.e., cutoff of complex amplitude with high frequency component or that of special AC components corresponding to spectrum of the discrete Fourier transform (DFT). It is possible to check the quality of reconstructed image from the viewpoint of the visual and intuitive appearance of three characters.



(1) Cutoff of high spatial frequency component
〈Use of DC+low AC components〉



(2) Cutoff of special spatial frequency component
〈Use of DC+low to high AC components〉

Fig. B Modified binary CGH and cutoff effect of reconstructed images