ALGEBRAIC STACKS OF POSITIVE CHARACTERISTICS

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ABSTRACT. In this article, the author studies the vanishing of cohomologies of smooth varieties over a field of positive characteristic. By using classifying algebraic stacks of gerbes of liftings to rings of Witt vectors of length two and Deligne-Illusie's theorem of degeneration of Hodge spectral sequence, we have some Kodaira-Kollár type vanishing theorems and Esnault-Viehweg type vanishing theorems.

1. INTRODUCTION

In characteristic 0, Kodaira-Nakano vanishing theorem is well-known. Kawamata found the application of divisors with coefficients of rational integers. Kollár and Esnault-Viehweg reduced vanishing theorems to Hodge theory. In characteristic $p > 0$, Deligne-Illusie([DI]) taught us that Frobenius maps work as well as Hodge theory. The hypothesis of Deligne-Illusie's theorems is that the ground field is liftable to rings of Witt vectors of length two. In general it is not the case but liftable as gerbe. We apply the general theory of algebraic stacks developed by Grothendieck, Giraud and Laumon ([Gir], [La2]).

2. PRELIMINARIES

Definition 2.1. Let X be a normal variety. Let $f : X \to S$ be a smooth morphism of varieties and D a divisor on X. Assume D has only simple normal crossings. If for each component C of D, the restriction $f : C \to S$ is smooth, then D is said to have only f-simple normal crossings.

We denote by $\mathbb{Z}_{(p)}$ the localization by the maximal ideal (p). Let $\mathrm{Div}(X)$ be the divisor group of X and $\mathrm{Div}(X) \otimes \mathbb{Z}_{(p)}$ the divisor group with coefficients in $\mathbb{Z}_{(p)}$. We denote by $[D]$ the integral part of $D \in \mathrm{Div}(X) \otimes \mathbb{Q}$, $\{D\} = D - [D]$ and $\lceil D \rceil = -[-D]$.

3. CHARACTERISTIC P > 0

Let X be a complete smooth variety of dimension d over a perfect field k of characteristic $p > 0$. When $X/\mathrm{Spec}\ k$ is liftable to $W_2(k)$, Deligne-Illusie's theory can be applied. In case $X/\mathrm{Spec}\ k$ is not liftable to $W_2(k)$, we use theory of stacks and reduce the degeneration of Hodge spectral sequence to the following Theorem by Deligne-Illusie ([DI]).

Theorem 3.1. Let k be a perfect field of characteristic $p > 0$, $S = \mathrm{Spec}\ k, S_2 = \mathrm{Spec}\ W_2(k)$ and X a smooth scheme over S.

To each flat lifting X_2 of X over S_2, there corresponds canonically an isomorphism of $D(X_2)$

$$\phi_{X_2} : \bigoplus_{i<p} \Omega^i_{X_2/S}[-i] \cong \tau_{<p} F_* \Omega^*_{X/S}$$

such that $\mathcal{H}^i \phi_{X_2} = C^{-1}$ for $i < p$. Moreover assume X is proper over S.

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(1) Hodge spectral sequence

$$E = H^{a+b}(X, \Omega_X^*) \Leftarrow E_1^{a,b} = H^b(X, \Omega_X^a)$$

degenerates at E_1 for $a + b < p$.

(2) if $\dim X \leq p$, the Hodge spectral sequence degenerates at E_1.

Now let k be a perfect field of characteristic $p > 0$, $S = \mathrm{Spec}\ k$, $S_2 = \mathrm{Spec}\ W_2(k)$ and X a proper smooth variety over k. Let R be the X-gerbe $R(X, S_2)$ of the liftings of X/S to S_2 and $B(R/X)$ the classifying X-champ which is algebraic of finite type with presentation $P \to B(R/X)$. Let $P' \to P$ be a projective birational morphism and P'/S quasi-projective by Chow's lemma. Let $P' \subset P''$ be an open immersion and P''/S projective.

Let Z the integral closure of X in P''. Let Y be the maximal radical extentions of X in Z. Let $f : Y \to X$ the structure morphism. Then there exists the least number r determined uniquely by X such that $R(X)^{p^{-\infty}} \cap R(Z) = R(X)^{p^{-r}} \cap R(Z)$. Since P''/Y is separable, we apply Theorem above to obtain the following Theorem.

Theorem 3.2. Let $D \in \mathrm{Div}(X) \otimes \mathbb{Z}_{(p)}$. Suppose the fractional part of D has the support in the normal crossing and D is numerically equivalent to zero. Then for any component C of the fractional part of f^*D, the homomorphism for $a + b > 2d - p$

$$H^b(Y, \Omega_Y^a\langle\lceil\{f^*D\}\rceil - C\rangle(C + [f^*D])) \to H^b(Y, \Omega_Y^a\langle\lceil\{f^*D\}\rceil\rangle(C + [f^*D]))$$

is injective and its dual

$$H^b(Y, \Omega_Y^a\langle\lceil\{f^*D\}\rceil\rangle(-C + [f^*D])) \to H^b(Y, \Omega_Y^a\langle\lceil\{f^*D\}\rceil - C\rangle([f^*D]))$$

is surjective for $a + b < p$. In particular,

$$H^b(Y, \mathcal{O}_Y(-C + [f^*D])) \to H^b(Y, \mathcal{O}_Y([f^*D]))$$

is surjective for $b < p$.

Corollary 3.3. Let $\mathcal{L}$ be an abundant invertible sheaf over X. Let B be a member of the linear system $|\mathcal{L}^{\otimes m}|$ for a number m prime to p. The homomorphism

$$H^b(X, \omega_X \otimes \mathcal{L}^{p^r}) \to H^b(X, \omega_X(B) \otimes \mathcal{L}^{p^r})$$

is injective for $b > d - p$. In particular, The homomorphism

$$H^{d-1}(X, \omega_X \otimes \mathcal{L}^{p^r}) \to H^{d-1}(X, \omega_X(B) \otimes \mathcal{L}^{p^r})$$

is injective.

Corollary 3.4. Let $r = r(X)$ be a number determined in the argument of Theorem above. Let $\mathcal{L}$ be an ample invertible sheaf over X. For $b > d - p$,

$$H^b(X, \omega_X \otimes \mathcal{L}^{p^r}) = 0.$$

In particular

$$H^{d-1}(X, \omega_X \otimes \mathcal{L}^{p^r}) = 0.$$

We would like to generalize theorems([DI]) to divisors with coefficients in $\mathbb{Z}_{(p)}$ ([M3]).

Theorem 3.5 (cf. [DI]). Let k be an algebraically closed field of characteristic $p > 0$. Let X be a smooth scheme over k. Let D be a $\mathbb{Z}_{(p)}$-divisor on X which is linearly equivalent to zero. Let A, B be divisors on X. Let $S = \mathrm{Spec}\ k$ and $\tilde{S} = \mathrm{Spec}\ W_2(k)$. Assume

(1) $\{D\}_{red} + A + B$ has only normal crossings.

(2) there exist a flat morphism $\tilde{X} \to \tilde{S}$ and a divisor $\tilde{D}$, $\tilde{A}$, $\tilde{B}$ such that the pull-back of $\tilde{X} \to \tilde{S}$ is $X \to S$ and $\tilde{D} + \tilde{A} + \tilde{B}$ is $\tilde{S}$-flat.

Then

$$F_*\Omega_X^a(A, B + \{D\})([D]) = \oplus\Omega_{X'}^a(A', B' + \{D'\})([D'])[-a]$$

$$F_*\Omega_X^a(A, B + \{D\})([D]) = \oplus\Omega_{X'}^a(A', B' + \{D'\})([D'])[-a]$$

Theorem 3.6 (cf. [DI]**).** Let k be a perfect field of characteristic $p > 0$, $S = \text{Spec } k$, $\tilde{S} = \text{Spec } W_2(k)$. Let X be a proper smooth scheme over k of pure dimension d. Let D be a $\mathbb{Z}_{(p)}$-divisor on X. Let $A+B$ be a divisor on X. Suppose $\{D\}_{red}+A+B$ has only normal crossings. Assume there exist liftings $\tilde{X}$ and $\tilde{D}$, $\tilde{A}$, $\tilde{B}$ such that $\tilde{X} \to \tilde{S}$ is flat and $\{\tilde{D}\}_{red} + \tilde{A} + \tilde{B}$ has only normal crossings on $\tilde{X}$ relative to $\tilde{S}$. Then

(1) Let C be a component of $\{D\}$. If D is numerically equivalent to zero, the next sequence is exact for all a, b

$$H^{b-1}(C, \Omega_C^a(A, B + \{D\}_{red} - C)([D])) \to$$
$$H^b(X, \Omega_X^a(A, B + \{D\}_{red})(-C + [D])) \to$$
$$H^b(X, \Omega_X^a(A, B + \{D\}_{red} - C)([D])) \to 0.$$

(2) If D is ample, the next cohomologies vanish for $a + b > \max(d_X, 2d_X - p)$

$$H^b(X, \Omega_X^a(A, B + \{D\}_{red})([D])) = 0.$$

Corollary 3.7. Let k be an algebraically closed field of characteristic $p > 0$. Let X be a projective smooth k-scheme of pure dimension d which is flatly liftable to $W_2(k)$. Let D be an ample $\mathbb{Z}_{(p)}$-divisor D on X. Assume $A + B + \{D\}_{red}$ has only simple normal crossings. Then one has

(1) for $a + b > \max(d, 2d - p)$

$$H^b(X, \Omega_X^a(A, B + \{D\}_{red})([D])) = 0$$

(2) for $a + b < \min(d, p)$

$$H^b(X, \Omega_X^a(B + \{D\}_{red}, A))(-[D])) = 0.$$

Corollary 3.8. Let k be an algebraically closed field of characteristic $p > 0$. Let X be a projective smooth k-scheme of pure dimension d which is flatly liftable to $W_2(k)$. Let D be a nef $\mathbb{Z}_{(p)}$-divisor D on X. Assume $A + B + \{D\}_{red}$ has only simple normal crossings and $\lceil D \rceil$ is big. Then one has

(1) for $b > \max(0, d - p)$

$$H^b(X, \omega_X(B + \lceil D \rceil)) = 0$$

(2) for $a + b < \min(d, p)$

$$H^b(X, \omega_X(B - \lceil D \rceil)) = 0.$$

Proof. We can take arbitrarily small $\varepsilon > 0$ and $\eta > 0$ such that

$$D = A + \varepsilon E - \eta\{-D\}$$

where A is ample. Hence there exists a certain $\varepsilon > 0$ such that $A + \varepsilon E$ is ample. Thus $D + \eta\{D\}$ is ample, the support of whose fractional part has only simple normal crossings.

Corollary 3.9. Let k be a perfect field of characteristic $p > 0$, $S = \mathrm{Spec}\, k$, $\tilde{S} = \mathrm{Spec}(W_2(k))$. Let X, Y be a projective smooth schemes over k of pure dimension. Let $f : X \to Y$ be a surjective morphism whose restriction is smooth over Y^o with $Y \setminus Y^o$ a normal crossing divisor. Assume there exists a lifting $\tilde{X}$ such that $\tilde{X} \to \tilde{S}$ is flat. Let L be an ample invertible sheaf over Y. Then

(1) $(f_*\omega_{X/Y})^{\otimes a} \otimes L^{\otimes b}$ is generically generated by global sections for arbitrary a and $b > \dim Y$.

(2) If $\omega_{X/Y}$ is f-semiample, $f_*\omega_{X/Y}^{\otimes \ell}$ is weakly positive for $\ell > 0$.

Definition 3.1 (Arithmetic variety). An arithmetic variety $\mathcal{X}$ is a projective flat regular scheme X over Spec $\mathbb{Z}$ the base change to Spec $\mathbb{C}$ of whose generic fibre has a Kähler metric compatible with complex conjugate action.

Theorem 3.10 ([M4]). Let X be a non singular variety and $f : X \to S$ a projective smooth morphism of analytic varieties. Let D be a $\mathbf{Q}$-divisor on X and D f-numerically equivalent to zero. Let A, B be divisors on X. Let D decompose itself into $D' + D''$ without common component. Assume $\{D\}_{red} + A + B$ has only f-simple normal crossings. Then Hodge-Deligne spectral sequence degenerates at E_1:

$$E_1^{a,b} = R^b f_* \Omega^a_{X/S}(A + \{D'\}_{red}, B + \{D''\}_{red})(\lceil D' \rceil + [D'']) \Rightarrow$$

$$R^{a+b} f_* \Omega^*_{X/S}(A + \{D'\}_{red}, B + \{D''\}_{red})(\lceil D' \rceil + [D'']).$$

4. NON FUJIKI MANIFOLDS

In this section, we obey the notation in ([Dm], [Dr], [Gr], [M5]). Owing to Hodge theory, most vanishing theorems are obtained in the category of Fujiki manifolds, i.e., those bimeromorphically equivalent to Kähler manifolds. Even outside the category of Fujiki, we have the following:

$$\delta = \delta_0 + [\Lambda_0, [\delta_0, \gamma]]$$

$$\Delta = [D, \delta] = [D, \delta_0] + [D, \Lambda_0, [\delta_0, \gamma]]$$

We proved the following theorem in ([M5]).

Theorem 4.1 (Commutativity). Let X be a compact Hermitian manifold and E a Hermitian line bundle over X. We denote by $c(E)$ the curvature of E. Δ_0 commutes with $ic(E)$, *i.e.*,

$$[\Delta_0, ic(E)] = 0.$$

The motivation of this note is to reconstruct the proof of Kollár and Esnault-Viehweg theory([Ko], [EV]) by Harmonic theory using

$$\begin{tikzcd} H^{a,b}(X,E) \arrow[r, "c_1(C)"] \arrow[d] & H^{a+1,b+1}(X,E) \arrow[r] & \mathcal{D}^{a+1,b+1}(X,E) \arrow[d] \\ H^{a,b}(C,E) & \arrow[l, "H"'] & \mathcal{D}^{a,b}(C,E) \end{tikzcd}$$

We give an example which explains the whole idea.

Theorem 4.2. Let X be a compact Kähler manifold and E a Hermitian line bundle over X. Assume the first Chern class $c_1(E^*) > 0$ everywhere. Let C be a non singular hypersurface such that there exists a $k > 0$

$$(E^*)^{\otimes k} \cong \mathcal{O}_X(C).$$

Then

$$H^{a,b}(X,E) \to H^{a,b}(C,E_C) \qquad \text{for all } a,\ b$$
$$u \mapsto u|_C$$

vanishes.

Proposition 4.3 (Demailly formula).

$$\Delta'' + [\Lambda, ic(E)] = \Delta'_\tau + T_\omega$$

where

$$T_\omega = [D' + \tau, \delta' + \tau^*] + [D'', [\Lambda, D']] - [\tau, \delta' + \tau] =$$
$$[\Lambda, [\Lambda, \frac{i}{2} d'd''\omega]] - [d'\omega, (d'\omega)^*]$$
$$\Delta_\tau = \Delta'_\tau + \Delta''$$
$$\Delta_\tau = [D + \tau, \delta + \tau^*], \Delta'_\tau = [D' + \tau, \delta' + \tau^*]$$

Theorem 4.4. Let X be a compact Hermitian manifold and E a Hermitian line bundle over X.

$$\Delta_0 = 2\Delta'' + [\Lambda, ic(E)]$$

Let C be a non singular hypersurface such that there exists a $k > 0, k \in \mathbb{R}$

$$c_1(E^*) = kc_1(C).$$
$$c_1(E) \leq 0$$

Then

$$H^{a,b}(X,E) \to H^{a,b}(C,E_C) \qquad \text{for all } a,\ b$$
$$u \mapsto u|_C$$

vanishes.

Proof.

$$[L_0, \Delta_0] = [L_0, [D, \delta_0] = [D, [\delta_0, L_0] + [\delta_0, [L_0, D]] = 0$$
$$4[\Delta'', \delta_0''] = [D - iD^c, \delta_0 + i\delta_0^c] =$$
$$[D, \delta_0] + [D^c, \delta_0^c] + i[D, \delta_0^c] - i[D^c, \delta_0] =$$
$$2[D, \delta_0] - 2i[\Lambda, D^2]$$
$$[D, \delta_0^c] = -[D, \delta_0^c] - 2[\lambda, D^2], \quad [D^c, \delta] = [D^c, \delta_0]^c = -[D, \delta_0^c]$$

$$\begin{tikzcd}
H^{a,b}(X,E) \arrow[r, "c_1(C)"] \arrow[d] & H^{a+1,b+1}(X,E) \arrow[r] & \mathcal{D}^{a+1,b+1}(X,E) \arrow[d] \\
H^{a,b}(C,E) & & \mathcal{D}^{a,b}(C,E) \arrow[ll, "H"']
\end{tikzcd}$$

Since $ic(E)$ is hermitian with respect to Hermitian metric of X, the eigen values of $ic(E)$ are real at any point x on X. We denote by $\alpha_1(x), \alpha_2(x), \alpha_3(x), \cdots, \alpha_n(x)$ these eigen values at

x. The local coordinates $(z_1, z_2, \cdots, z_n)$ with center at x are taken so that $(\frac{\partial}{\partial z_1}, \frac{\partial}{\partial z_2}, \cdots, \frac{\partial}{\partial z_n})$ form an orthonormal system in the stalk $T_x X$ of the vector bundle. We have

$$ic(x) = \frac{i}{2} \sum_{1 \leq j \leq n} \alpha_j(x)\, dz_j \wedge\, d\bar{z}_j$$

For $v \in H^b(X, \Omega^a_X(E))$,

$$v = \sum_{|A|=a, |B|=b} v_{A,B} e \otimes dz_A \wedge d\bar{z}_B$$

$$[\Lambda, ic(E)]v = \sum_{A,B} (\alpha_N - \alpha_A - \alpha_B) v_{A,B} e \otimes dz_A \wedge d\bar{z}_B$$

$$[ic(E), [\Lambda, ic(E)]]v = \sum_{j \in N} 2\alpha_j^2 v_{A,B} e \otimes dz_j \wedge d\bar{z}_j \wedge dz_A \wedge d\bar{z}_B$$

Since

$$[\Delta'', ic(E)] + [[\Lambda, ic(E)], ic(E)] = 0,$$

we have

$$\langle [\Delta'', ic(E)]v, ic(E)v \rangle + \langle [[\Lambda, ic(E)], ic(E)]v, ic(E)v \rangle = 0.$$

Here

$$\langle [[\Lambda, ic(E)], ic(E)]v, ic(E)v \rangle = \sum_{j \in N} 2\alpha_j^3 \langle v_{A,B} e \otimes dz_j \wedge d\bar{z}_j \wedge dz_A \wedge d\bar{z}_B, v_{A,B} e \otimes dz_j \wedge d\bar{z}_j \wedge dz_A \wedge d\bar{z}_B \rangle$$

and

$$\langle [\Delta'', ic(E)]v, ic(E)v \rangle \geq 0.$$

It implies that

$$c_1(E)v = 0.$$

Example 1. Let H be a classical Hopf surface.

$$H = \mathbf{C}^2 / \langle \frac{1}{2} \rangle$$

This Hopf surface H has an elliptic fibration over $\mathbb{P}_1$. Let E_1, E_2 be the two elliptic curves on H over $0, \infty \in \mathbb{P}_1$, respectively. The restriction map

$$H^1(H, \Omega^1(-E_i)) \to H^1(E_1, \Omega(-E_i))$$

vanishes. They form the exact sequence:

$$H^1(H, \Omega^1(-E_i)) \to H^1(E_1, \Omega^1(-E_i)) \to H^2(H, \Omega^1\langle E_i \rangle(-2E_i))$$

$$H^{1,1}(H) = 0, H^{1,1}(E_i) = \mathbb{C}, Pic(H) \cong \mathbb{C}^*$$

$$H^{0,1}(H) = \mathbb{C}, H^{1,0}(H) = H^{2,0}(H) = H^{0,2} = 0$$

$$H^1(H, \mathcal{O}(-E_i)) \to H^1(E_1, \mathcal{O}(-E_i))$$

vanishes. They form the exact sequence:

$$H^1(H, \mathcal{O}(-E_i)) \to H^1(E_1, \mathcal{O}(-E_i)) \to H^2(H, \mathcal{O}(-2E_i)).$$

Theorem 4.5. Let X be a compact Hermitian manifold and E a Hermitian line bundle over X.

$$\Delta_\tau + T_\omega = 2\Delta'' + [\Lambda, ic(E)]$$

For any section $u \in H^b(X, \Omega_X^a(E))$ such that

$$\langle T_\omega u, u\rangle \geq 0,$$

then one has

$$\begin{tikzcd} H^b(X,\Omega_X^a(E)) \arrow[r] \arrow[d] & H^b(X,\Omega_X^{a+1}(E)) \arrow[d] \\ H^b(X,\Omega_X^a\langle C\rangle(E)) \arrow[r] & H^b(X,\Omega_X^{a+1}\langle C\rangle(E)) \end{tikzcd}$$

$$H^b(X,\Omega_X^{a+1}\langle C\rangle(E)) \xrightarrow{\text{restriction}} H^b(C,\Omega_C^a(E|_C))$$

The composition map

$$H^b(X,\Omega_X^a(E)) \longrightarrow H^b(C,\Omega_C^a(E|_C))$$

associates to u zero.

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