Simplified compression and reconstruction techniques of digital images by means of discrete wavelet transforms

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Abstract

The main aim of wavelet transform is to provide an intuitive and visual representation of digital signals or image data from the viewpoint of time-frequency or space-scale relation, respectively.

Many aspects of digital images : the effect of resolution level, visual appearance of compressed and reconstructed images, and image fidelity between two different images are discussed in connection with different filter coefficients such as D2, D4, and D8 which are used in scaling equations.

A fidelity degree of reconstructed images may be visually evaluated using a display result of the difference image between an original image and its reconstructed image, although the commonly used error criterion is a quantitative measure of the root-mean-squared error or PSNR.

1. Introduction

As pointed out by Chui [1], a wavelet has been a very popular topic of conversations in many scientific and engineering meetings these days. Some regard wavelets as a new mathematical subject or a basic idea for representing functions, and others think it as a new technique for time-frequency signal analysis or image processing. All of them are right, since wavelet is a versatile tool with mathematical content and has a great potential for many applications.

A theory of wavelets can be seen as a common framework for many techniques that had been developed independently in various fields such as mathematics, signal processing, multiresolution, subband coding, etc. In particular, the wavelet transform is of interest for the analysis of non-stationary signals, because it provides another powerful way in place of the short-time Fourier transform.

Recently, Rioul and Vetterli [2] explained the relation between multidimensional filter banks and wavelets in connection with wavelets and signal processing. Meyer [3] published an introductory and distinctive book (translated and revised by R. D. Ryan) containing a historical perspective of wavelets. The concept of multiresolution analysis plays a central role in Mallat's algorithm for the decomposition and reconstruction of images. But, an idea related to multiresolution analysis for digital images, namely the Laplacian pyramid scheme, was published by Burt and Adelson [4] for the first time. Wavelet transforms can be seen as a variation of Laplacian pyramid decomposition. In the discrete case, they are equivalent to a

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logarithmic filter bank with the added constraint of regularity on the low-pass filter. Daubechies [5] studied an orthonormal basis of compactly supported wavelets, and carried out a review of multiresolution analysis. She showed how orthonormal bases of wavelets can be constructed starting from a multiresolution analysis. Yoshida et al. [6] reported an application of the wavelet transform to automated detection of clustered microcalcifications, and a fast processing technique of wavelet transforms recently.

In this paper, simplified compression and reconstruction techniques of digital images are treated by contrast with JPEG technology based on DCT (Discrete Cosine Transform) [7]. Many aspects of digital images: the effect of resolution level, visual appearance of compressed and reconstructed images, and image fidelity between two different images are discussed in connection with different filter coefficients such as D2, D4, and D8 in the scaling function and wavelets.

A fidelity degree of reconstructed images may be visually evaluated using a display result of the difference image between two similar images, although the commonly used error criterion is a statistical measure of the root-mean-squared error or PSNR.

2. Two scale difference equations and application of wavelets to image processing

The concept of multi-resolution approximation of functions was introduced by Meyer [8] and Mallat [9], and provides a powerful framework to understand the wavelet decompositions. The multi-resolution idea is very intuitive. A better approximation of a signal may be obtained by adding the details specified as Wj to the approximate signal Sj at the specified resolution level j.

The approximate signal Sj stands for the smoothing image, and the details contain the three different sub-images: horizontal, vertical, and diagonal images. See Fig. 1 and Fig. 2.

Two scale difference equations may be defined as follows [9].

Scaling function:

$$\begin{aligned}\phi(x) &= \sum l(k) \cdot \phi(2x-k)\\ &= \sum_k C_k \cdot \phi(2x-k) \qquad (1)\end{aligned}$$

Wavelet:

$$\begin{aligned}W(x) &= \sum h(k) \cdot \phi(2x+k-N+1)\\ &= \sum_k (-1)^k \cdot C_k \cdot \phi(2x+k-N+1) \qquad (2)\end{aligned}$$

where, $l(k)$; $h(k)$: coefficients for low-pass and high-pass filters

C_k: wavelet coefficient

N: even number (filter length or tap)

Note that the special factor: $(-1)^k$ in Eq. (2) is necessary in order to transform the role of low-pass filter into that of high-pass filter.

The coefficients of the low-pass filter are derived from the special conditions: (1) conservation of area, (2) accuracy of expansion, (3) orthogonality of scaling function. The coefficients of the high-pass filter have relation to those of the low-pass filter.

Table 1 shows the necessary and sufficient conditions for calculating the wavelet coefficients and a typical result of C_k with the even number: N=2, 4 and 8. There are three different categories of conditions.

1. conservation of area: the sum of the coefficients must always equal 2.
2. condition of accuracy: the expansion of a length of signal in a finite number of wavelets must be achieved as closely as possible.
3. condition of orthogonality: the scaling function and wavelets generated from the special filter coefficients are orthogonal to

Table 1 Necessary and sufficient conditions for wavelet coefficients C_k with even number N

(i) Conservation of area
$\Sigma C_k = 2$

(ii) Accuracy of expansion
$\Sigma(-1)^k k^m C_k = 0$
$m = 0, 1, 2, \ldots\ldots, N/2-1$

(iii) Orthogonality
$\Sigma C_k \cdot C_{k+2m} = 0,\ m \neq 0$
$\Sigma (C_k)^2 = 2$

[Notes] $\Sigma \triangleq \sum_{k=0}^{N-1}$

Scaling function : $\phi(x) = \Sigma C_k \cdot \phi(2x-k)$
Wavelet : $W(x) = \Sigma(-1)^k C_k \cdot \phi(2x+k-N+1)$

Haar wavelet $N=2$

from (i) $c_0 + c_1 = 2$
from (ii) $c_0 - c_1 = 0$
from (iii) $c_0^2 + c_1^2 = 2$
whose solution is just
$c_0 = c_1 = 1$

D 4 wavelet after Daubechies $N=4$		*D 8 wavelet* $N=8$
from (i)	$c_0 + c_1 + c_2 + c_3 = 2$	
from (ii)	$c_0 - c_1 + c_2 - c_3 = 0$	$c_0 = 0.3258$
	$-c_1 + 2c_2 - 3c_3 = 0$	$c_1 = 1.0109$
from (iii)	$c_0c_2 + c_1c_3 = 0$	$c_2 = 0.8922$
	$c_0^2 + c_1^2 + c_2^2 + c_3^2 = 2$	$c_3 = -0.0396$
whose solution is		$c_4 = -0.2645$
$c_0 = (1+\sqrt{3})/4$	$c_1 = (3+\sqrt{3})/4$	$c_5 = 0.0436$
$c_2 = (3-\sqrt{3})/4$	$c_3 = (1-\sqrt{3})/4$	$c_6 = 0.0465$
		$c_7 = -0.0150$

The coefficients : C_K used in this paper are derived by multiplying the results : $h(n)$ published by Daubechies by $\sqrt{2}$.

each other.

The basic function $\phi(x)$ is a dilated (horizontally) version of $\phi(2x)$. The function $\phi(x)$ generated from a unit box (unit height and length) is called the scaling function. For example, putting $N=2$, and $k=0$ to 1 in Eq. (1), the simple scaling function with the Haar coefficients: $C_0 = C_1 = 1$ may be derived using the three conditions shown in Table 1. C's are numerical constant (generally some positive and some negative).

$$\begin{aligned}\phi(x) &= C_0\phi(2x) + C_1\phi(2x-1)\\ &= 1\times\phi(2x) + 1\times\phi(2x-1)\end{aligned} \tag{3}$$

A wavelet $W(x)$ is derived from the corresponding scaling function by taking differences. Note that there are two different definitions as for the coefficients of wavelets as follows.

$$W(x) = \sum_k (-1)^k C_k \cdot \phi(2x+k-N+1)$$
$$W(x) = -C_1 \cdot \phi(2x) + C_0 \cdot \phi(2x-1) \quad ; N=2$$

or (4)

$$W(x) = \sum_k (-1)^k C_{1-k} \cdot \phi(2x-k)$$
$$W(x) = C_1 \cdot \phi(2x) - C_0 \cdot \phi(2x-1) \quad ; N=2 \tag{5}$$

where, C_k : coefficient of non-zero
($k = 0, 1, 2, \ldots.$)

The same coefficients are used as for the definition of $W(x)$, but in reverse order in comparison with Eq. (4) and with alternate terms having their signs changed from minus to plus or from plus to minus. The number of these coefficients is always even.

In order to apply the wavelet decomposition to multi-dimensional signal (e.g., digital images), multi-dimensional extensions of wavelets are required. An obvious way to do this is to use the notion of separable wavelets obtained from the products of one-dimensional scaling function and wavelet.

$$\begin{aligned}\phi(x,y) &= \phi(x)\cdot\phi(y)\\ W_H(x,y) &= \phi(x)\cdot W(y)\\ W_V(x,y) &= W(x)\cdot\phi(y)\\ W_D(x,y) &= W(x)\cdot W(y)\end{aligned} \tag{6}$$

These functions are orthogonal to each other with respect to integer shifts. The function $\phi(x, y)$ is a separable two-dimensional scaling function corresponding to a spatial low-pass filter. On the other hand, the functions $W_H(x, y)$, $W_V(x, y)$, and $W_D(x, y)$ are wavelets, and fill the role of spatial high-pass filters, respec-

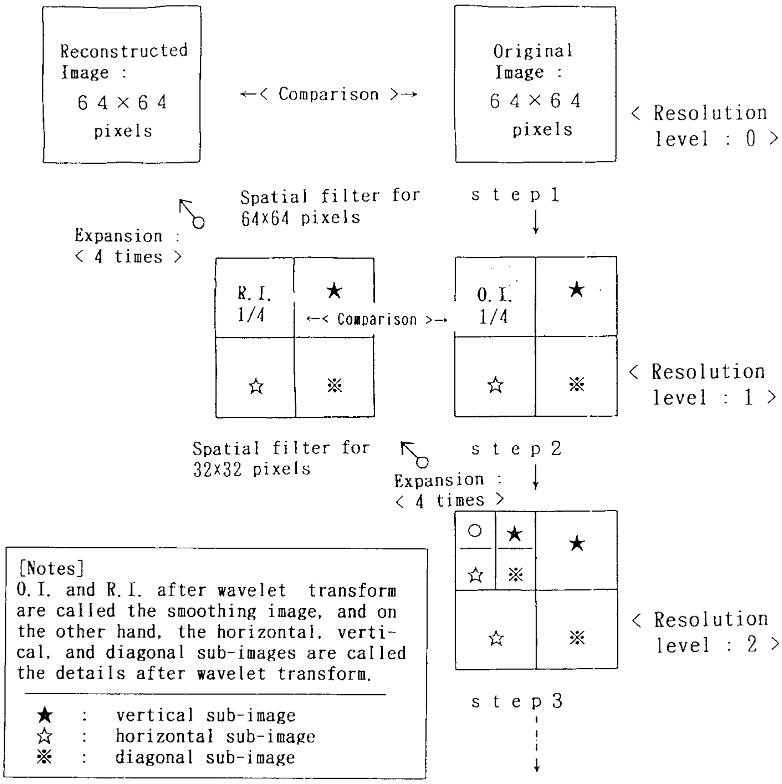


Fig. 1 Image decomposition and synthesis using wavelet transforms

tively.

The typical coefficients for composing the scale function and wavelets are derived from multiplying the results h(n) published by Doubechies by constant parameter $\sqrt{2}$. From the viewpoint of a two-dimensional filtering operation, two-dimensional wavelet transform may be simply implemented without directly applying the one-dimensional splitting algorithm to the horizontal and vertical sequences, namely rows and columns of an image, on condition that an idea of the mixed or united square transform matrix is introduced.

3. Formulation of wavelet transforms using matrix operations

A pair of discrete transforms and matrix expressions with separable transformation kernels of digital images which are composed of M×N pixels may be formulated as follows [11].

· Forward transform :

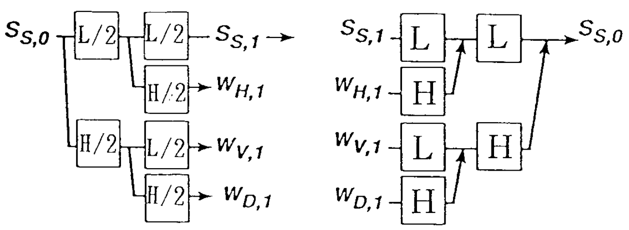
$$F(u,v)=\sum_{m}\sum_{n} T_F(m,n:u,v)\ f(m,n)$$
$$[F]=[T_V]\times[f]\times[T_H]^t \tag{7}$$

Inverse transform :

$$f(m,n)=\sum_{u}\sum_{v} T_I(m,n:u,v)\ F(u,v)$$
$$[f]=[T_V]^t\times[F]\times[T_H] \tag{8}$$

where, f(m, n) ; [f] : original image
F(u,v) ; [F] : transformed image (transformed coefficients)
T_F ; T_I : transformation kernel or basic function
[T_H] ; [T_V] : horizontal and vertical transformation matrixes
t : symbol of transposed matrix

Fig. 1 shows an example of the process of image decomposition and synthesis using the wavelet transform. A few symbols such as「☆」, 「★」,「※」stand for the specified domains of the horizontal, vertical and diagonal sub-images,



(a) Wavelet decomposition from level 0 to 1 (b) Wavelet reconstruction from level 1 to 0

Fig. 2 Decomposition and reconstruction of digital images by means of spatial low/high pass filters

respectively.

An original image may be divided into four sub-images composed of smoothed, horizontal, vertical, and diagonal parts as a result of the wavelet transform. The smoothed sub-image may be recursively divided into four small sub-images. On the contrary, reconstructed image may be simply synthesized from a set of the four sub-images.

Table 2 Algorithms for decomposition and its reconstruction of digital images using matrix operation

(a) Decomposition from level j to j+1

$$S_{S,\,j+1} = (1/2)\cdot L_j S_{S,\,j}\cdot (1/2)\cdot (L_j)^T$$
$$W_{H,\,j+1} = (1/2)\cdot L_j S_{S,\,j}\cdot (1/2)\cdot (H_j)^T$$
$$W_{V,\,j+1} = (1/2)\cdot H_j S_{S,\,j}\cdot (1/2)\cdot (L_j)^T$$
$$W_{D,\,j+1} = (1/2)\cdot H_j S_{S,\,j}\cdot (1/2)\cdot (H_j)^T$$

(b) Reconstruction from level j to j−1

$$S_{S,\,j-1} = (L_j)^T S_{S,\,j} L_j + (H_j)^T W_{H,\,j} L_j + (L_j)^T W_{V,\,j} H_j + (H_j)^T W_{D,\,j} H_j$$

[Notes]

$S_{S,\,j}$: smoothing (or original) image at level j

$W_{H,\,j}$; $W_{V,\,j}$; $W_{D,\,j}$: horizontal, vertical, and diagonal images at level j, after wavelet transforms, respectively

L_j ; H_j : spatial low pass and high pass filters composed of $2^{n-1}\times 2^n$ components at level j

$(\ )^T$: transposed matrix

n : integer ($n\geqq 1$)

Fig. 2 shows a successive process of wavelet decomposition and reconstruction of images. The boldface letters such as 「L」 and 「H」 stand for the use of the low-pass and high-pass filters, respectively. The first subscript letter means the specified domains of smoothing, horizontal, vertical and diagonal sub-images, respectively. The second subscript means the specified resolution level for wavelet transforms, on the assumption that zero means the start resolution level of an original image (corresponding to the smoothing image).

Table 2 shows an algorithm for the decomposition and reconstruction of digital images using the matrix operations in connection with Fig. 2. The first and second subscripts mean the same role as shown in the case of Fig. 2. The four different sub-images at the resolution level: j+1 may be transformed from the smoothing image at the resolution level: j without applying the matrix operations in Eq. (9).

Note that the transform matrixes are equivalent to two types of coefficients : $\ell(k)$ and $h(k)$ for the scaling function and the wavelet, respectively.

The filter L acts like an integrator and H as a differentiator. The pairs {L, H} are also called quadrature mirror filters.

A pair of two-dimensional discrete wavelet transform and its inverse transform may be performed directly by means of the matrix operations, on the assumption that the matrix size of an original image data is equal to the n-th power of 2.

Image decomposition with four small sub-images

$$W = \{(1/2)\times T\times F\}\cdot[(1/2)\times T]^t = T_n\times F\times T_n^{\,t} \qquad (9)$$

Table 3 Matrix expressions for spatial low & high pass filters based on Haar coefficients : n=2, and matrix expression for synthetic spatial filter

$$L_1 = [1 \quad 1]$$
$$H_1 = [1 \quad -1]\ ;\ [-1 \quad 1]$$

$$L_2 = \begin{bmatrix} 1 & 1 & & \\ & & 1 & 1 \end{bmatrix}$$
$$H_2 = \begin{bmatrix} 1 & -1 & & \\ & & 1 & -1 \end{bmatrix} ; \begin{bmatrix} -1 & 1 & & \\ & & -1 & 1 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 1 & 1 & & & & & & \\ & & 1 & 1 & & & & \\ & & & & 1 & 1 & & \\ & & & & & & 1 & 1 \end{bmatrix}$$
$$H_3 = \begin{bmatrix} 1 & -1 & & & & & & \\ & & 1 & -1 & & & & \\ & & & & 1 & -1 & & \\ & & & & & & 1 & -1 \end{bmatrix} ;$$
$$\begin{bmatrix} -1 & 1 & & & & & & \\ & & -1 & 1 & & & & \\ & & & & -1 & 1 & & \\ & & & & & & -1 & 1 \end{bmatrix}$$

$$T_n = 1/2 \begin{bmatrix} Ln \\ \hline Hn \end{bmatrix} = 1/2 \begin{bmatrix} 2^{n-1} \times 2^n \\ \hline 2^{n-1} \times 2^n \end{bmatrix} \quad (n \geqq 1)$$

[Note] When the wavelet transforms, i. e., wavelet decompositions are carried out, the normalization factor : 1/2 is necessary for the transform matrix : Tn in advance.

Image reconstruction or synthesis

$$F = \{T^t \times W\} \times T \qquad (10)$$

where, F : original image

W : wavelet image (a set of four small sub-images)

T : united square matrix composed of a combination of low-pass and high-pass filters

Tn : normalized transform matrix

It is possible to synthesize a united transform matrix by applying the special expansion coefficients for the scaling function and wavelets. The united square matrix may be composed of two types of filter coefficients, i. e., a combination of the spatial low-pass and high-pass filter coefficients. An idea of simple matrix operations called the orthogonal transforms may be used in order to carry out a discrete wavelet and its inverse transforms recursively. As a result. a simplified compression and reconstruction of digital images with $2^n \times 2^n$ pixels may be effectively performed at the specified resolution level.

Note that the two same constant parameter : 1/2 is necessary for the normalization of the square matrix : T in the case of image decomposition. The wavelet image contains a set of four small sub-images, i. e., smoothed, horizontal, vertical, and diagonal images at the specified resolution level, when carrying out directly the matrix operations in Eq. (9).

Table 3 shows an example of the matrix composition for spatial low-pass and high-pass filters based on the Haar coefficients: n=2, i. e., the coefficients of the scaling function in Eq. (1), and (3). In this study, an idea of the united square matrix is used for the matrix operations formulated in Eq. (9) and Eq. (10). These matrix operations may be applied recursively or repeatedly by considering the size of united square matrix at the specified resolution level.

4. Image Compression and Its Fidelity Evaluation

An important role of image transformation or decomposition is to reduce an extra-component or a dynamic range of the signal (or data) in order to eliminate redundant information that can be compressed more effectively. For example, DCT or DWT is often used in an original image to create the transformed and modified images in connection with the digital image compression and coding problems [12].

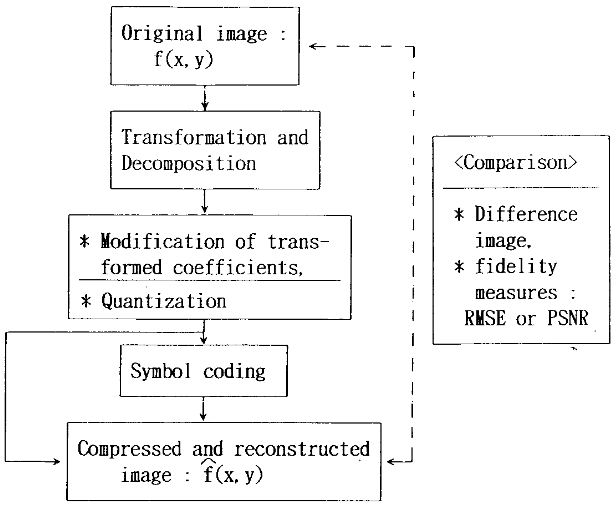


Fig. 3 Simple framework for lossy compression and reconstruction of digital image data

Fig. 3 shows a simple framework for a lossy compression of digital image data. An orthogonal transform technique such as DFT, DCT and DWT may be used to transform and/or decompose the original image data. An adaptive spatial filter is proposed to modify and/or alter transformed coefficients in the case of digital image compression and reconstruction based on DFT and DCT [13]. Quantization process is very useful for a bit allocation and a symbol coding.

The primary difference between lossy and lossless techniques of digital data is the inclusion of quantization in lossy techniques owing to modification of transformed coefficients. By quantizing the data, the number of possible output symbols may be reduced. In this study, the symbol encoding process such as Huffman coding or arithmetic coding is omitted in order to simply evaluate the visual appearance of compressed and reconstructed images, and the image fidelity.

Many statistical and quantitative measures are proposed in order to evaluate the image fidelity between two similar images [14].

It is important to note that a lower RMSE (root-mean-squared error) or a equivalently higher PSNR (peak signal-to-noise ratio) does not necessarily imply a higher subjective reconstructed image quality. The error measures defined as RMSE and PSNR do not always correlate well with perceived image quality. Denoting the original $N \times N$ image by $f(x, y)$ and the compressed-reconstructed image by $\hat{f}(x, y)$, RMSE is given by the following expression.

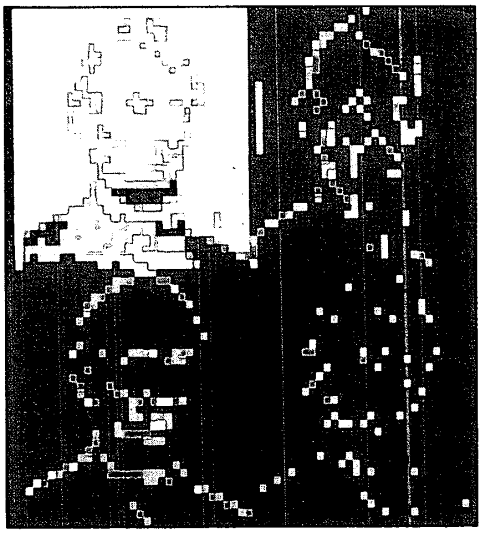
$$\mathrm{RMSR} = (1/N)\sqrt{\sum_{x}\sum_{y}[f(x,y) - \hat{f}(x,y)]^2} \quad (11)$$

The related measure of PSNR in dB is computed for a 5 bit (0-31 gray levels) image as follows.

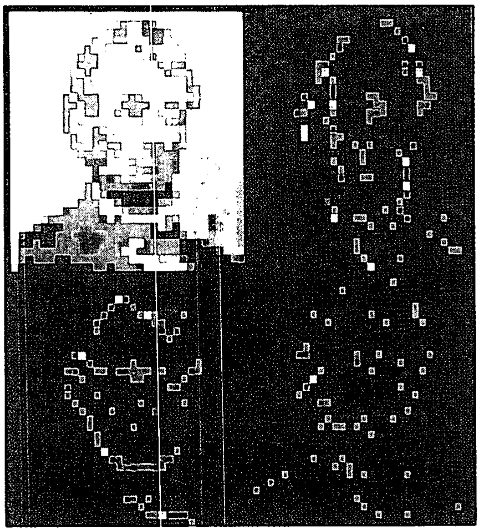
$$\mathrm{PSNR} = 20 \times \log_{10}(31/\mathrm{RMSE}) \quad (12)$$

If only the diagonal sub-image data is deleted in Fig. 1, the reconstructed image visually resembles the original image. The difference between two similar images is evaluated directly from the difference image. It is possible to discuss the image fidelity from the viewpoint of statistical and quantitative evaluation.

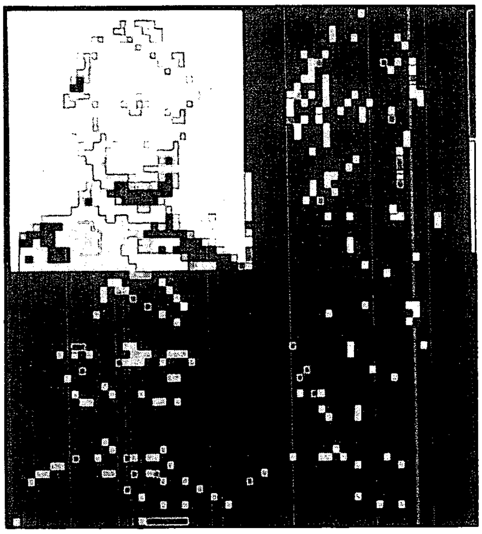
Fig. 4 shows a result of the wavelet transform by means of five different types of spatial filters at the resolution level : 1. The coefficients for the scaling function and wavelet, i. e., D2, modified D2, D4, modified D4, D8 coefficients are used. As regards the numerical values for the D2, D4 and D8 coefficients, see Table 1 or reference [5]. The coefficients : Cn used in this paper are derived by multiplying the results : h(n) published by Daubechies by $\sqrt{2}$. The results are demonstrated in quasi -colors. The specified colors : black, blue, red, green, and gray correspond to gray levels : 0, 1, 2, 3, 4-15, respectively. The upper left part of each image, namely reduced and smoothed sub -image corresponds to a quarter of the original



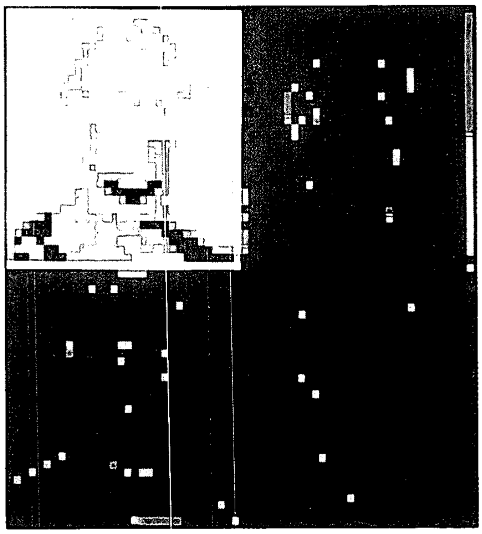
(a) D2 filter coefficients



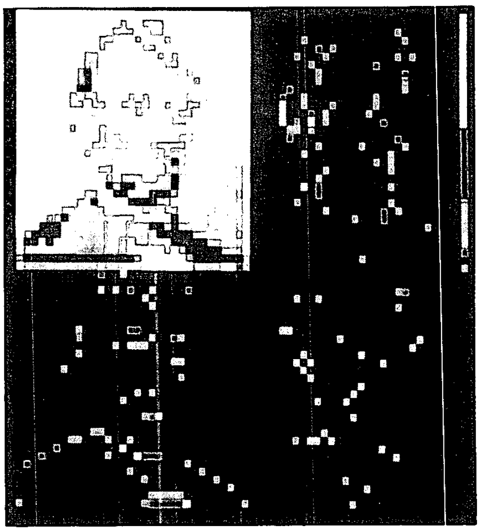
(b) Modified D2 filter coefficients



(c) D4 filter coefficients

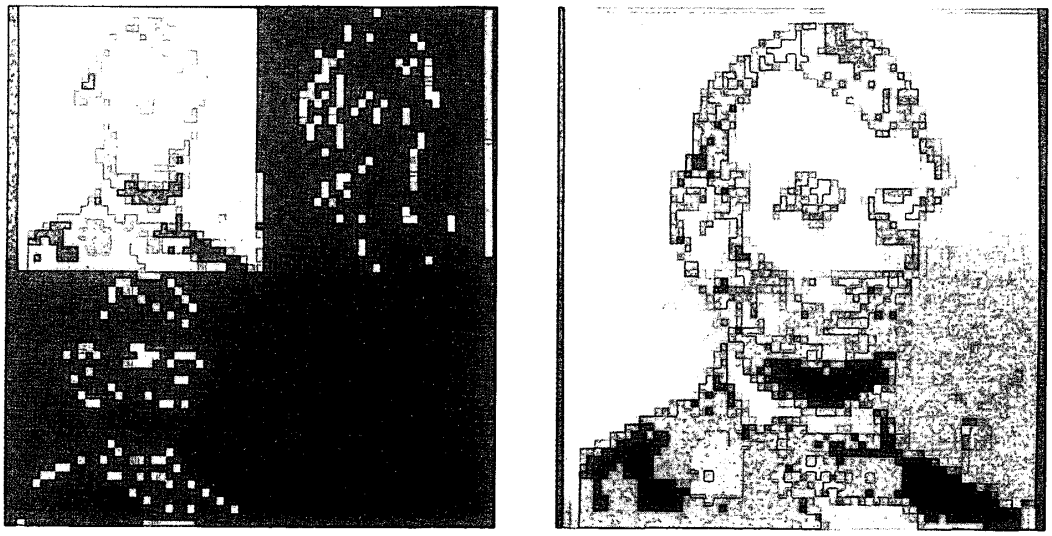


(d) Approximate D4 filter coefficients

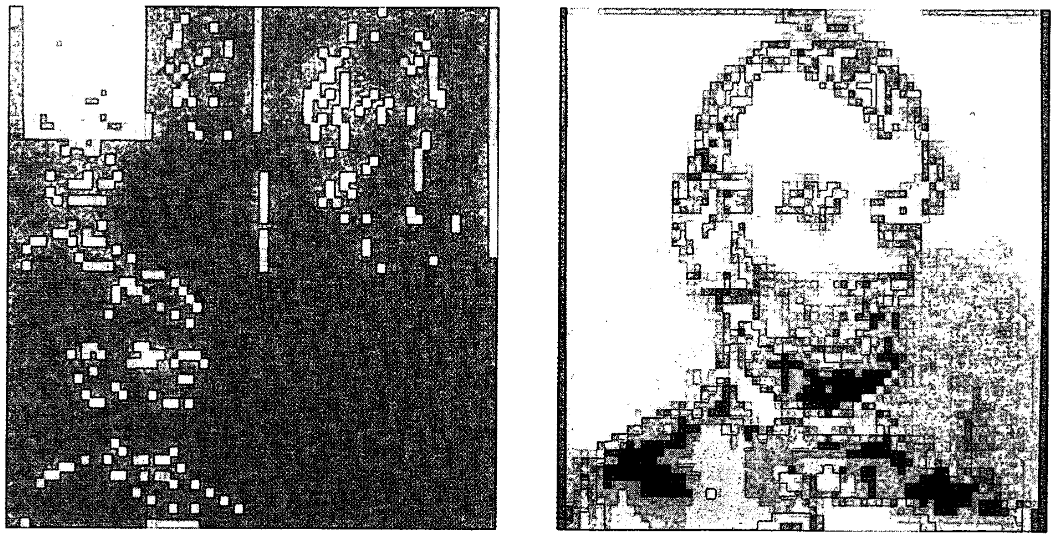


(e) D8 filter coefficients

Fig. 4 Wavelet transforms at the resolution level : 1 using five types of filter coefficients



(a) Resolution level : 1



(b) Resolution level : 2

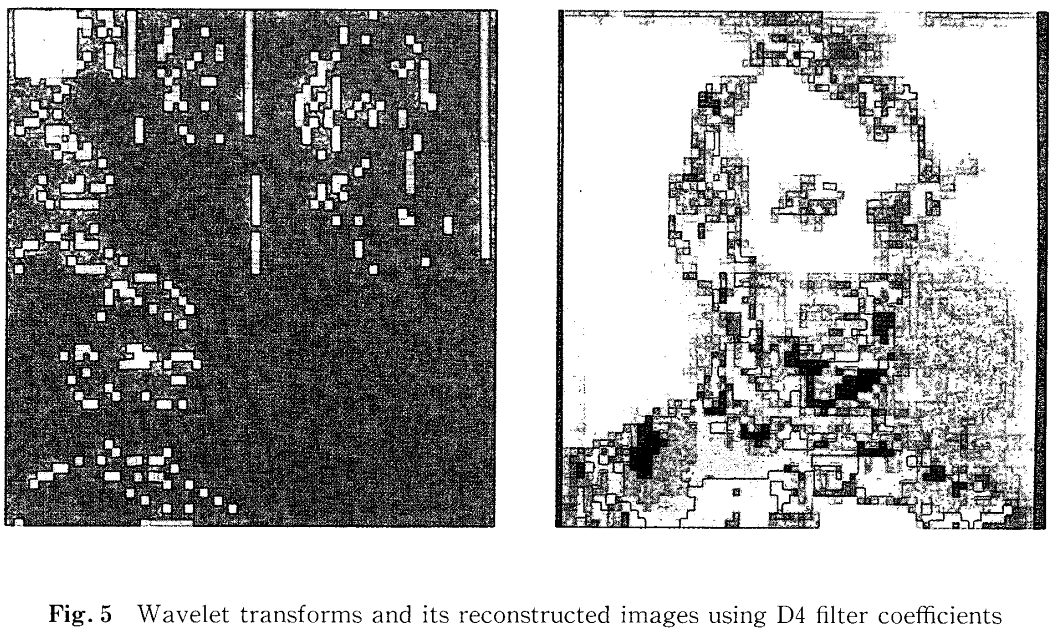


Fig. 5 Wavelet transforms and its reconstructed images using D4 filter coefficients

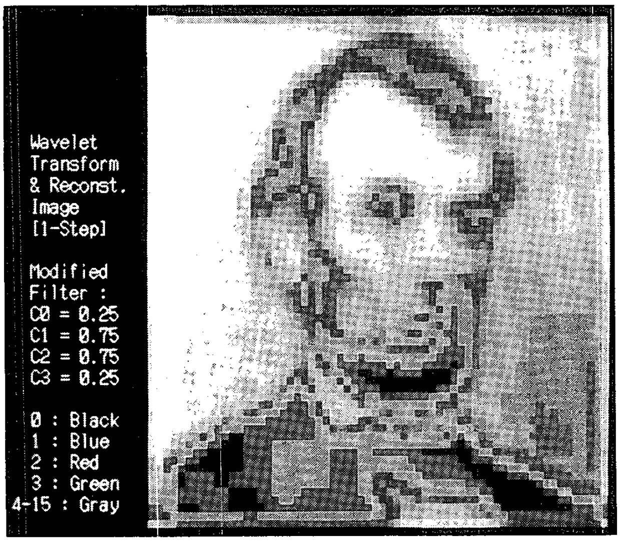
image. The aspect of smoothed images is different from each other and is degenerated compared with that of the original image. The lower right part is called diagonal sub-image, and may be obtained as a result of spatial filterings of horizontal and vertical directions.

Fig. 5 shows a result of the wavelet transforms and compressed/reconstructed images by using the D4 coefficients. The resolution level is different from each another, and the diagonal sub-images are deleted in the case of the wavelet transform. The smoothed sub-images are much degenerated at the resolution level: 3 which is composed of 8×8 pixels. But, the visual appearance of compressed/reconstructed images is very good in contrast with the results, i.e., smoothed sub-images in Fig. 4 to some degree.

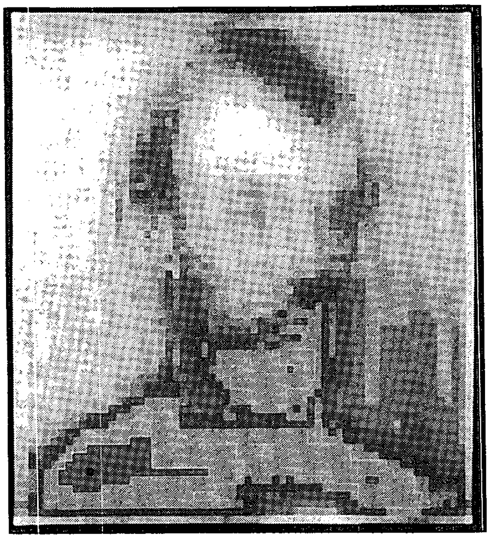
Fig. 6 shows an image reconstructed by using the modified coefficients similar to D4 filter coefficients. Note that the number of specified resolution level for reconstruction differs from each another.

It is possible to discriminate the visual appearance of reconstructed image. In order to demonstrate clearly the appearance of reconstructed images, a display of results is demonstrated in quasi-color. But, the visual appearance of compressed/reconstructed images is very degenerate in contrast with the results of Fig. 5.

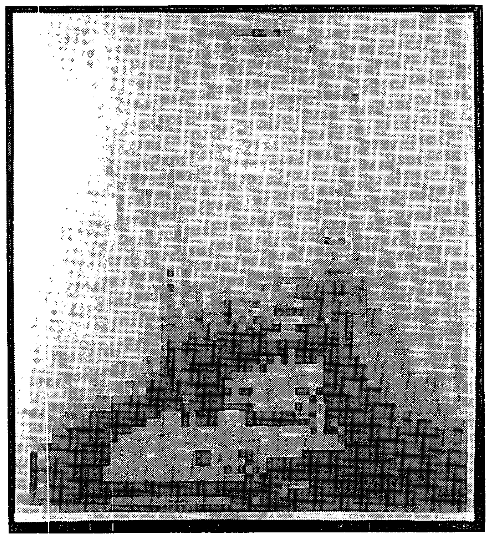
Fig. 7 demonstrates a result of two original images and difference image 〈original image − reconstructed image〉 displayed from the resolution level 1 by using the standard D2 filter coefficients. The middle part of the difference image is displayed by subtracting the image reconstructed by the four sub-images from the



(a) Resolution level: 1

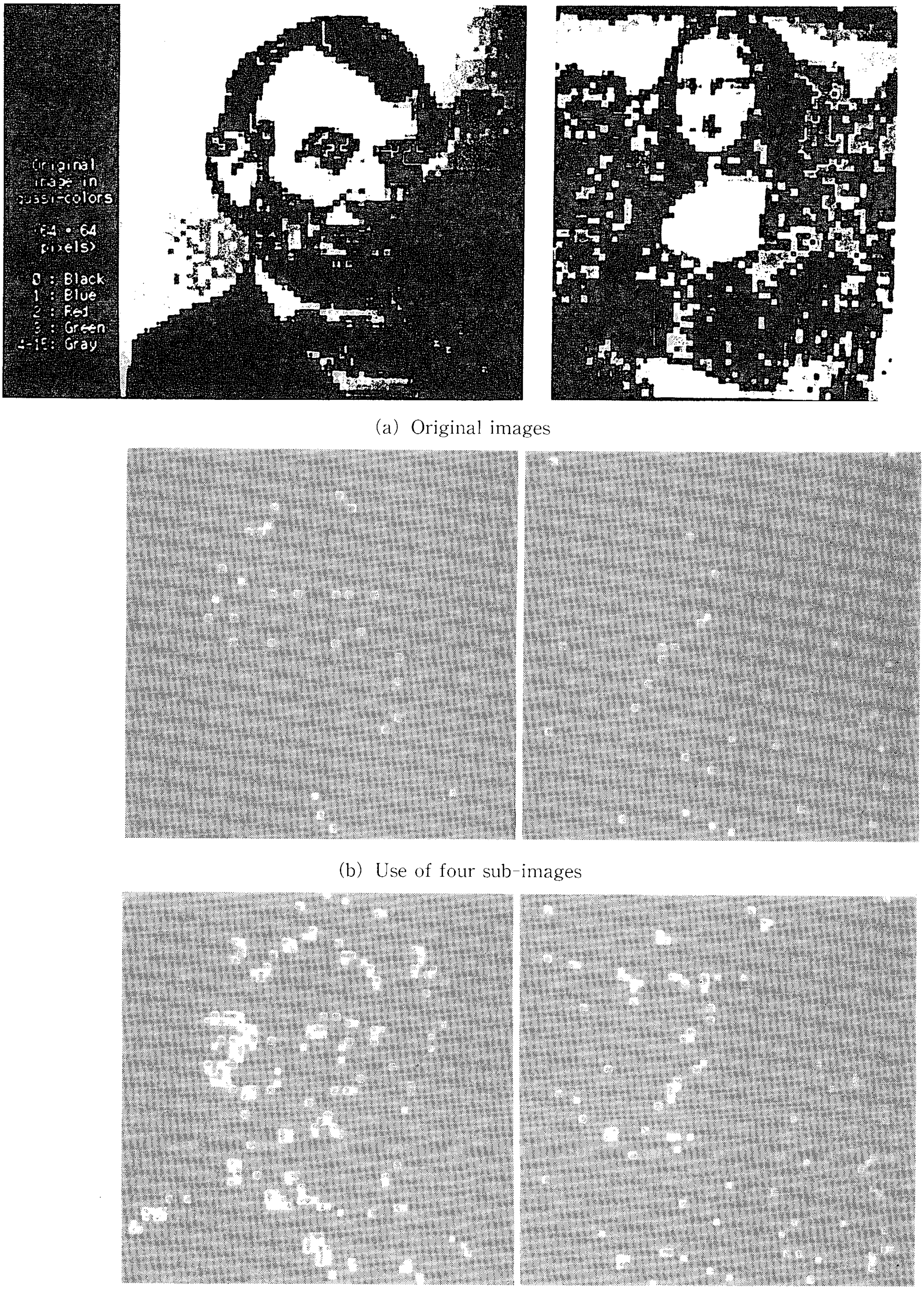


(b) Resolution level: 2



(c) Resolution level: 3

Fig. 6 Compressed and reconstructed images using approximate D4 filter coefficients

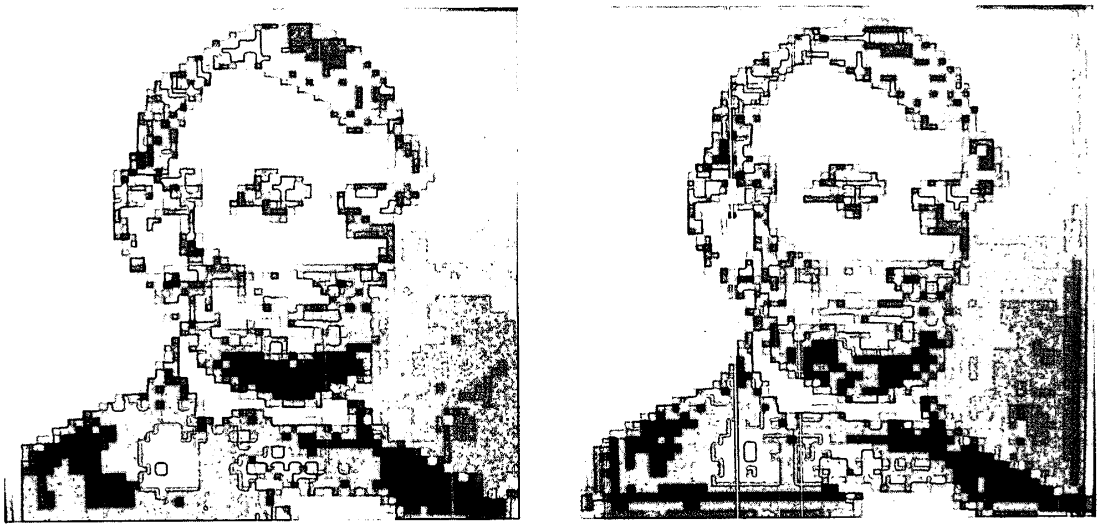


(a) Original images

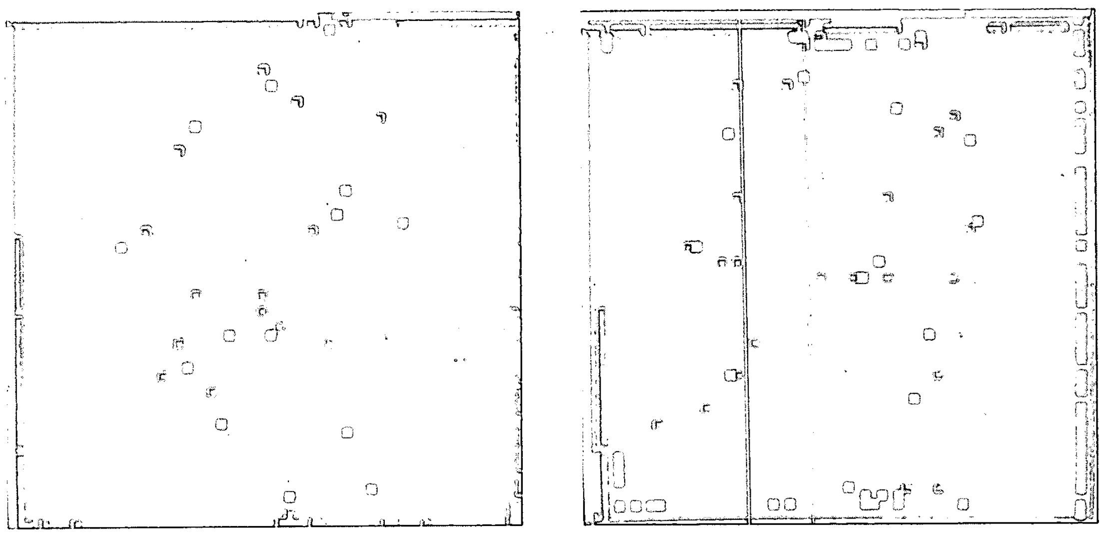
(b) Use of four sub-images

(c) Use of three sub-images
(deletion of diagonal sub-image)

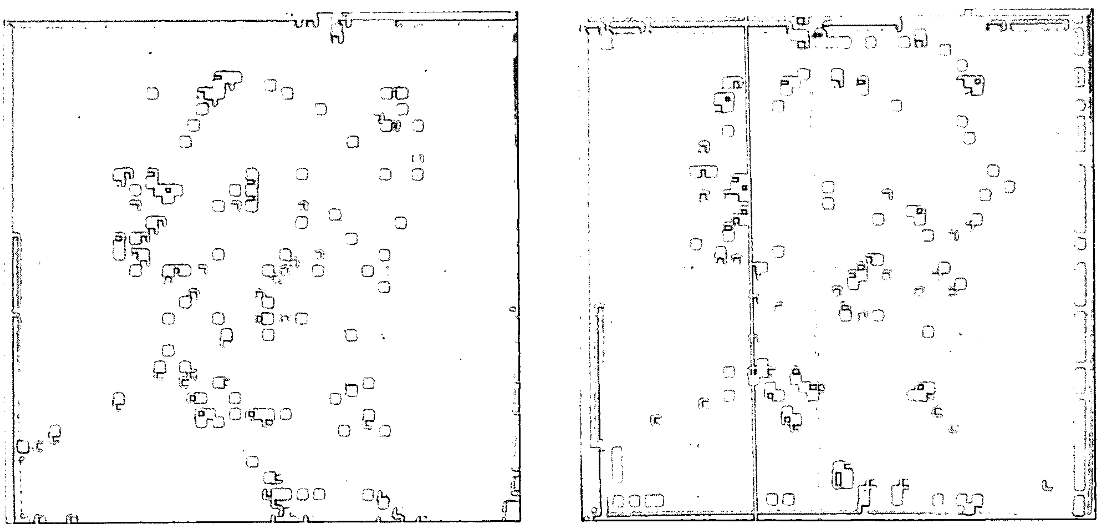
Fig. 7 Original images in quasi-colors and difference image at the resolution level : 1 using D2 filter coefficients



Compressed and recomstructed images

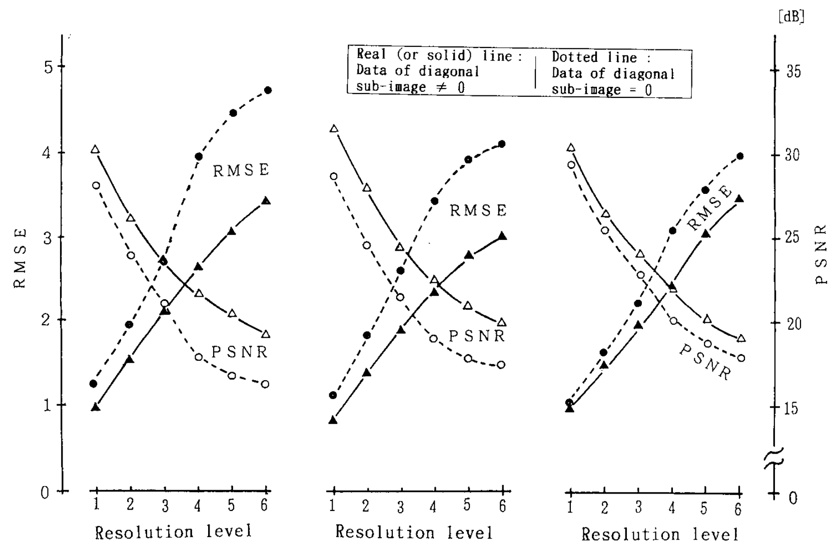


(a) D4 filter coefficients



(b) D8 filter coefficients

Fig. 8 Compressed/reconstructed images and difference images at the resolution level : 1 using three sub-images



(a) Lincoln image (b) Boy image (c) Mona Lisa image

Fig. 9 Image fidelity : RMSE and PSNR calculated using D2 filter coefficients

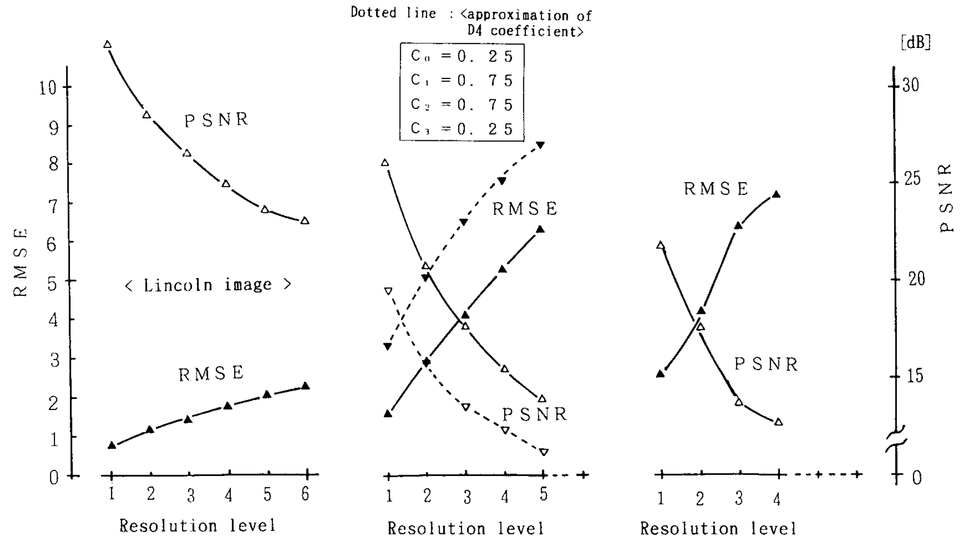
original image. On the other hand, the difference images at the lower part are displayed by using the reconstructed (or smoothed) image by the three sub-images, i. e., deletion of the diagonal sub-image. When the following condition : $-1 \leqq$ D. I. = O. I. − R. I. $\leqq 1$ holds, cyan color is used in order to discrimiante two similar images intuitively. The conditions : D. I. < -1 and D. I. > 1 hold, red and green colors are used, respectively.

The effect of deleling the diagonal sub-images is investigated. As a result, the outline of the original image is stressed, because there exists a dominant difference between the two images.

Fig. 8 demonstrates a result of the reconstructed image on the condition that the diagonal sub-image at the resolution level 1 is deleted, and the difference images obtained by two different filter coefficients : D4 and D8 are displayed in quasi-color. The middle parts show the difference images in the case of the use of four sub-images, and the lower parts show the difference images in the case of three sub-images, i. e., truncation of diagonal sub-image. In comparison with the results in Fig. 7, the quasi-colors are clear and dominant at the part of the border.

Fig. 9 shows the evaluation results of statistical measures indicating image fidelity : RMSE and PSNR at six different resolution levels. The real line demonstrates the fidelity of reconstructed images. On the other hand, the dotted line demonstrates the fidelity of compressed and reconstructed images obtained by the deletion of the diagonal sub-images at the six different resolution levels. As the resolution level increases, the visual appearance of reconstructed images becomes degenerate. The value of PSNR decreases the dynamic range of the statistical measures in contrast with that of RMSE.

Fig. 10 shows the evaluation results of image fidelity : RMSE and PSNR. The differences of statistical measures owing to the three different filter coefficients are demonstrated at several resolution levels. Note that the dotted lines are



(a) D2 coefficient (b) D4 coefficient (c) D8 coefficient

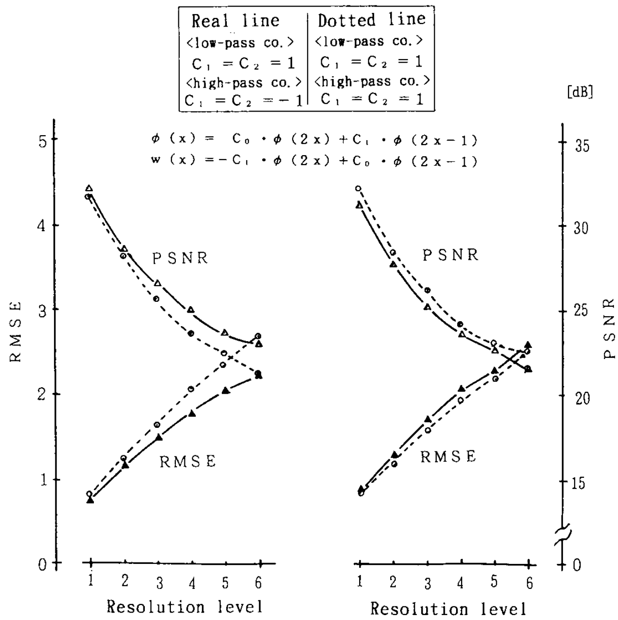
Fig. 10 Image fidelity : RMSE and PSNR obtained by three different filter coefficients

equivalent to the use of the modified D4 filter coefficients in Fig. 5. In this case, image fidelity of the reconstructed images becomes lower in comparison with the results of the standard D4 filter coefficients used in Fig. 6.

Fig. 11 shows the evaluation results of image fidelity, on condition that the two types of coefficients for two-scale relations of the scaling function $\phi(x)$ and its corresponding wavelet $W(x)$ are specified in advance. Note that the value of fidelity measure differs from each other to some degree. But, it is difficult to discern the appearance of the reconstructed image from that of the original image in the case of low resolution level, although there are a little differences in the numerical value of the statistical measures.

5. Conclusions

Simplified compression and reconstruction of digital images were carried out by applying a notion of multiresolution analysis (MRA), i. e., the two-dimensional wavelet transform techniques.



(a) Lincoln image (b) Mona Lisa image

Fig. 11 Image fidelity : RMSE and PSNR obtained by D2 and its modified filter coefficients

To sum up, the main conclusions are as follows.

(1) The discrete wavelet transform and its inverse wavelet transform may be effectively carried out using the real-valued united transform matrices in place of the discrete cosine and Fourier transforms.

(2) By means of the orthogonal transform matrices composed of two types of coefficients : a set of spatial low-pass and high-pass filters, the wavelet transform may be recursively performed until the specified resolution level.

(3) The visual appearance of the compressed and reconstructed images is affected by not only the type of specified filter coefficients but the resolution level for wavelet transforms.

(4) The use of D2 filter coefficients in contrast with that of D4 and D8 filter coefficients is very effective from the viewpoint of the fast simplified compression and reconstruction of digital images.

(5) It is possible to evaluate intuitively the image fidelity between two similar images from the display results of difference images without using statistical measures such as RMSE and PSNR.

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<Appendix>

Though a wavelet analysis and its related fields have a relatively new subject, there are already some unique books which contain a lots of reviews and papers. These distinguished books contribute to both wavelet theories and its applications.

[a] Yves Meyer Ed. : Wavelets and Applications, Masson/Springer-Verlag. (1992)

[b] Mary Beth Ruskai et al., Eds. : Wavelets and

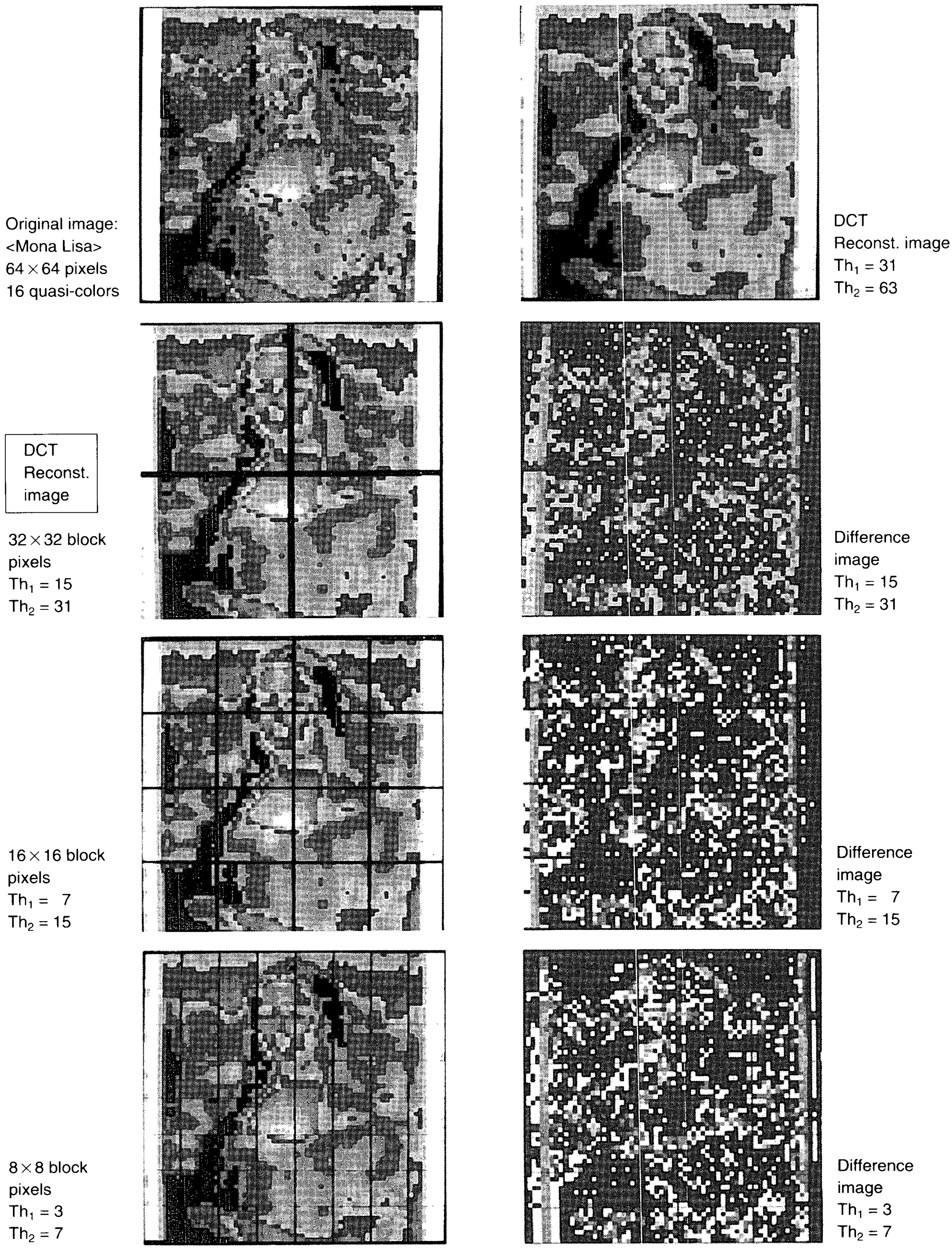

Fig. A-1 Quasi-color representations of reconstructed and difference images composed of four kinds of block pixels

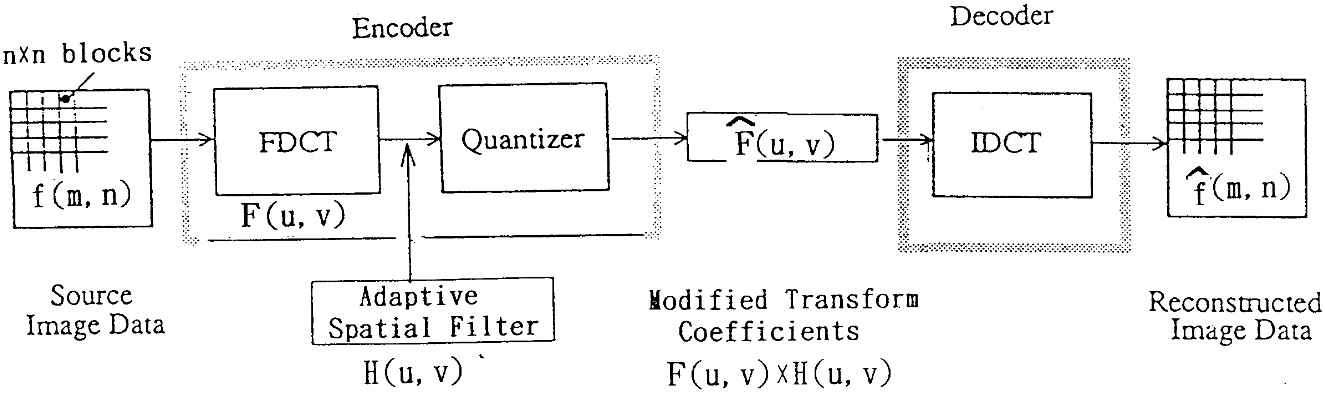

Fig. A-2 Simplified process for image compression by means of adaptive spatial filter

Their Applications, Janes and Bartlett Publishers, (1992)

[c] Tom H. Koornwinder Ed.: Wavelets: An Elementary Treatment of Theory and Applications, World Scientific. (1993)

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Fig. A-1 shows a representation of the original and reconstructed images in quasi-color, and the difference image composed of 16×16 block pixels, respectively. The original image data are composed of 64×64 pixels with 32 gray-levels, and may be used as the simple test data for many image processing techniques and their applications. The two reconstructed images are demonstrated by using a series of DCT (Discrete Cosine Transform) filtering processing, and IDCT (Inverse DCT) processes shown in **Fig. A-2**. The two kinds of threshold values: Th1=7 and Th2=15 are specified for the adaptive spatial filter shown in *Fig. A-3〈C〉*.

As a means for intuitively understanding the aspect of block distortion or degradation of a reconstructed image in contrast to an original image, a display technique based on the difference image between the original and the reconstructed image is used extensively. As a result, the degree of similarity or difference between the two may be simply checked irrespective of various numerical evaluation measures. The quasi-color representation of the difference image is very useful for intuitive evaluation of image quality, i.e., image fidelity.

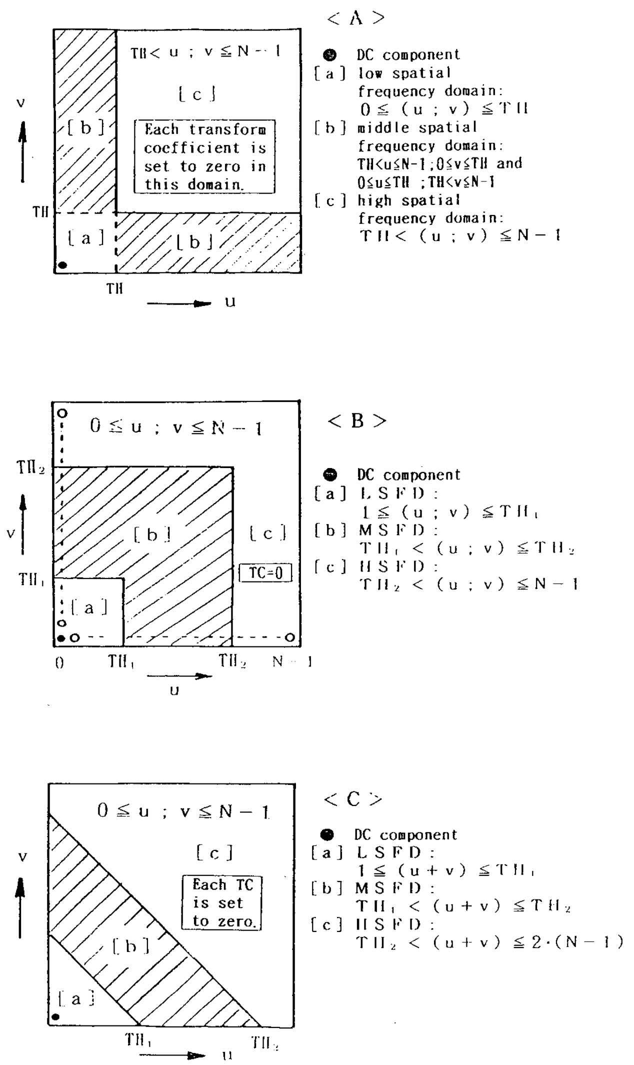
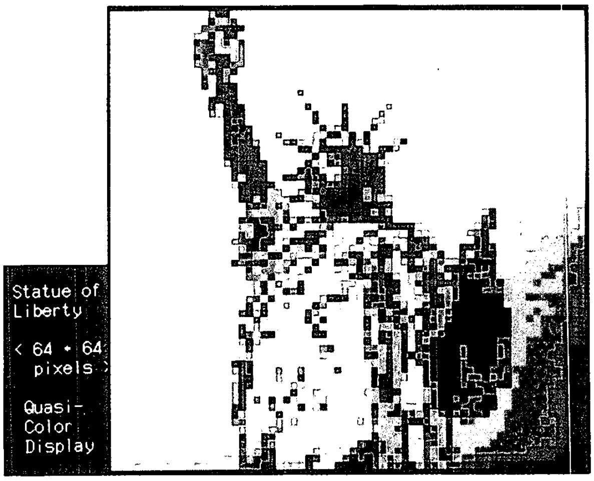
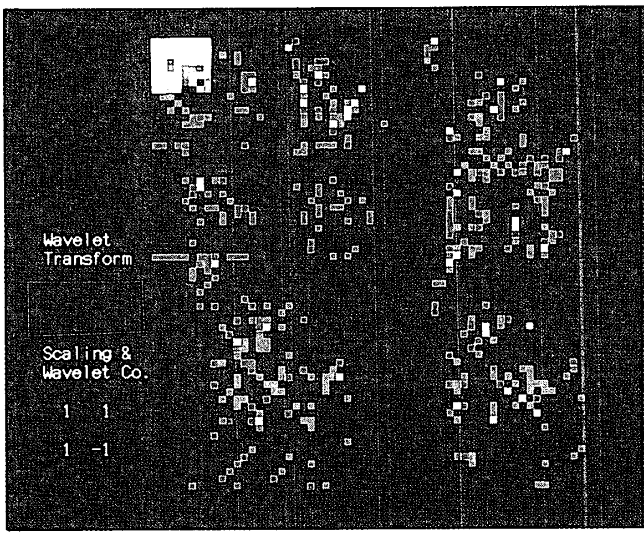


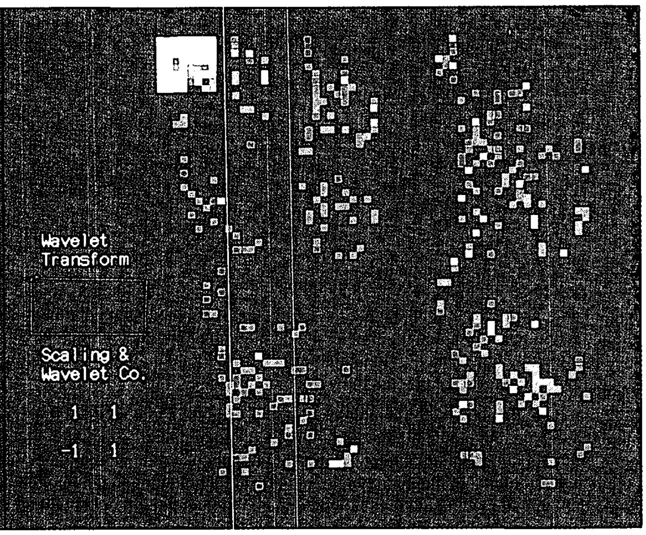
Fig. A-3 Division of spatial frequency domain

Fig. A-3 shows the three different types of division of the spatial frequency domains: u and v. Note that a DC component is demonstrated in the black mark: ● at



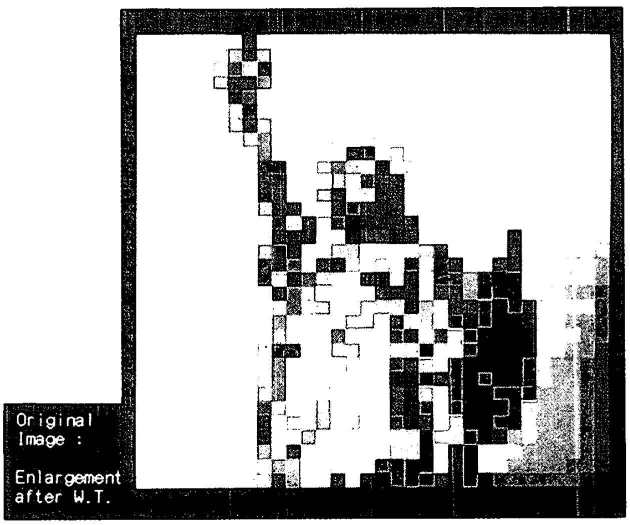
(a) Original Image : Statue of Liberty

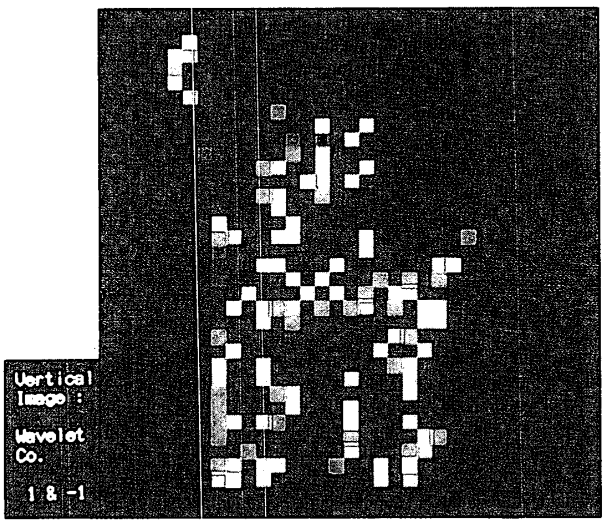




(b) Wavelet transform at R. L. = 1 to 3 by using Wavelet co. : 1 & −1

(c) Wavelet Transform at R. L. = 1 to 3 by using wavelet co. : −1 & 1





(d) Reconstructed smoothing sub-image from R. L. = 1

(e) Wavelet vertical sub-image at R. L. = 1

Fig. A-4 Original image and various images based on wavelet transfrom the origin (lower left side).

Fig. A-4 shows an example of the original image called the Statue of Liberty and its wavelet transform based on D2 coefficients. Note that a quasi-color representation is used for the visual and/or intuitive demonstration of horizontal and vertical sub-images.

Fig. A-4(a) shows the original image composed of 64×64 pixels in 16 quasi-colors.

Fig. A-4(b) and (c) shows the results of wavelet transform at the resolution level: 1 to 3. We cannot discern the small smoothing subimage with 8×8 pixels, i. e., a part of the upper left at the resolution level: 3. There is a decided difference between the two regarding the wavelet coefficients corresponding to a set of the spatial high-pass filter. A reduced smoothing sub-image and three kinds of different diagonal sub-images have the same display results each other except for the horizontal and vertical sub-images. The results of the wavelet transform in *Fig. A-4(b) and (c)* have the same number of pixels in contrast with the original image data. As the resolution level for the wavelet transform increases, the frequency or ratio of the gray levels such as 0, 1, 2, 3 etc. (which correspond to black, blue, red, green colors, respectively) becomes high or large gradually. It is possible to carry out an effective compression of digital image data together with a bit coding technique which may be used with a view to decreasing the amount of information.

Fig. A-4(d) shows the result of enlarged smoothing sub-images after carrying out the wavelet transform at the resolution level: 1. This image is different from an original image reconstructed by using a set of four kinds of sub-images, i. e., smoothing, horizontal, wertical and diagonal sub-images. In comparison with Fig. A-4(a), the size of each pixel is 4 times larger than that of the original pixel. As a result, a very blurred smoothing sub-image is demonstrated.

Fig. A-4(e) shows an enlarged vertical sub-image at the resolution level: 1, i. e., a part of upper right side in Fig. 4-A(e). A borderline of the original image or smoothing sub-image may be demonstrated simply and roughly without applying a Laplacian filter for an accurate detection of the border-line.

[Note]
The negative values of expansion coefficients are replaced by the zero value at each resolution level, when displaying the three sub-images except for the smoothing images corresponding to the reduced and blurred original image in this study.