Position and Attitude Estimation from a Image Sequence of a Circle

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ABSTRACT

A method to estimate the position and attitude of a helicopter with respect to the landing site from a image sequence of a heliport is presented. The method use the circle of the heliport marking as the visual cue. The projection of the circle on the successive image taken by on board camera will change, therefore a Kalman filter can be build for the recursive estimation. The method needs to know just there is a circle ; The size of the circle is not necessary. The result of the experiment on the synthetic data shows the method works well under several assumptions.

1. INTRODUCTION

Unmanned flight of a helicopter is strongly expected because it can perform various critical tasks such as rescue and security operations, traffic monitoring, mountain fire fight, and inspection of power transmission lines, without exposing a human pilot to danger. In autonomous flight, the estimation of the parameters that relate to the vehicle state should be done by the instruments instead of a human pilot. On-board GPS (Global Positioning System)/INS (Inertial Navigation System) or ground-based beacon systems are generally used for this purpose. However, as these systems are designed for long range, low precision flight, the measurement accuracy does not meet the requirements in certain missions. Recently, the utilization of the images taken by on-board camera has been investigated to make up the defects of the above-mentioned navigation systems [1], [2], [3], [4], [5].

The group of Carnegie-Mellon University has been working on the object tracking task by controlled flight of a helicopter, in which they attempt to get more precise estimate of the position and the velocity of a helicopter from the displacement of consecutive images of the ground in cooperating with attitude data provided by Gyroscope [1], [2]. NASA Ames Research Center use the image data to detect and locate the obstacles in nap-of-the-Earth flight mode [3], [4]. In their approach, the vehicle state is computed by INS and provided to the estimation process

The landing approach is also the task which requires the accurate state estimation. We must know precisely the relative position and attitude between a vehicle and a desired landing site to land a helicopter safely. A human pilot

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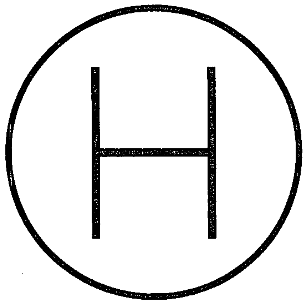


Fig. 1 Heliport Marking

perform this by observing the ground scene while both of the vehicle and the heliport need to be fully equipped for automatic landing. The images can be expected to provide much information for this task.

In this study, we try to estimate the position and the attitude of a helicopter from a image sequence of a heliport. A circle is selected as the visual cue because every heliports in Japan ought to be marked by a circle and a character 'H' as shown in Fig. 1. We assume that we know there is a circle but don't know the size of the circle. We also assume that the linear and angular velocity of a helicopter can be obtained by on-board sensors.

2. IMAGE GEOMETRY

We assume that the camera is mounted at the center of gravity of the helicopter and oriented with the viewing axis along the helicopter's longitudinal body axis. Fig 2 shows the viewing geometry of the camera. Let the image plane be perpendicular to the viewing axis at a distance f from the origin of the camera-axis (focal length). Then the image coordinate of a point on the ground are represented in camera coordinate (x_S, y_S, z_S) as

$$x = f x_s / z_s \qquad y = f y_s / z_s \tag{1}$$

3. STATE ESTIMATION FROM A IMAGE SEQUENCE OF A CIRCLE

According to the image geometry mentioned above, a circle on the ground is projected into an ellipse when observed by an on-board camera of a helicopter. The equation of a ellipse is given by

$$Ax^2 + 2Bxy + Cy^2 + 2fEy + f^2F = 0 \tag{2}$$

in image coordinate (x, y) and 6 coefficients, A, B, C, D, E, F are represented with $s = (s_1, s_2, s_3)^t$, the vector from the center of the circle to the helicopter, $n = (n_1, n_2, n_3)^t$, the surface normal vector of the plane on which the circle lies, and r, the radius of the circle as

$$A = s_2^2(1-n_3^2) + s_3^2(1-n_2^2) + 2s_2s_3n_2n_3 - r^2n_1^2$$
$$B = s_3^2n_2n_1 + s_1s_2(1-n_3^2) - s_2s_3n_3n_1 - s_3s_1n_2n_3 - r^2n_1n_2$$
$$C = s_3^2(1-n_1^2) + s_1^2(1-n_3^2) + 2s_3s_1n_1n_2 - r^2n_2^2$$
$$D = s_2^2n_3n_1 - s_1s_2n_2n_3 - s_2s_3n_1n_2 + s_1n_3(1-n_2^2) - r^2n_3n_1$$

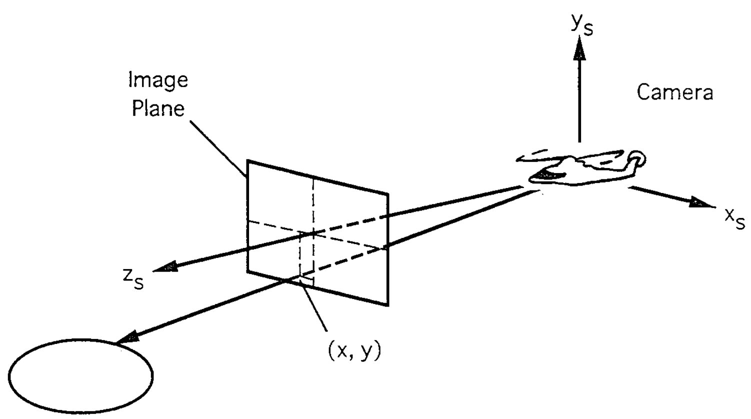


Fig. 2 Image geometry in the camera coordinate system

$$E = s_1^2 n_2 n_3 - s_3 s_1 n_1 n_2 - s_1 s_2 n_3 n_1 + s_2 s_3 (1 - n_1^2) - r^2 n_2 n_3$$
$$F = s_1^2 (1 - n_2^2) + 2 s_1 s_2 n_1 n_2 + s_2^2 (1 - n_1^2) - r^2 n_3^2 \tag{3}$$

Because only the ratios between the coefficients have physical significance, we divide both side of Eq. (3) by C, the coefficient of y^2 term, to get

$$A'x^2 + 2B'xy + y^2 + 2fD'x + 2fE'y + f^2F' = 0 \tag{4}$$

$$\begin{aligned} A' &= A/C \\ B' &= B/C \\ D' &= D/C \\ E' &= E/C \\ F' &= F/C \end{aligned} \tag{5}$$

Now, the problem may be formulated as follows. "Estimate s, n and r by observing 5 coefficients of ellipses (A′, B′, D′, E′, F′) on the projected images." The actual ellipse coefficients will be diferent from the true value due to the noise in the sensor and errors introduced by the preprocessing (image enhancement and feature detection). Let (A_m', B_m', D_m', E_m', F_m') be the measured coefficients of the ellipse in the image, such that

$$\begin{aligned} A'_m(t) &= A'(t) + n_{A'}(t) \\ B'_m(t) &= B'(t) + n_{B'}(t) \\ D'_m(t) &= D'(t) + n_{D'}(t) \\ E'_m(t) &= E'(t) + n_{E'}(t) \\ F'_m(t) &= F'(t) + n_{F'}(t) \end{aligned} \tag{6}$$

where $n_{A'}$, $n_{B'}$, $n_{D'}$, $n_{E'}$, and $n_{F'}$ represent noise of imaging system. We assume these are independent scalar white noise processes with standard deviations $\sigma_{A'}$, $\sigma_{B'}$, $\sigma_{D'}$, $\sigma_{E'}$, and $\sigma_{F'}$, respectively. In vector notation, measured or actual ellipse coefficients can be represented as

$$Z(t) = h(t) + \zeta_z(t) \tag{7}$$

where

$$h(t) = (A', B', D', E', F')^T$$
$$\zeta_z(t) = (n_{A'}, n_{B'}, n_{D'}, n_{E'}, n_{F'})^T$$

and

$$R \equiv \mathrm{cov}(\zeta_z) \equiv \begin{bmatrix} \sigma_A'^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_B'^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_D'^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_E'^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_F'^2 \end{bmatrix}$$

The observed ellipse will move and change in the shape on the image as the helicopter flies, therefore measurements of ellipse coefficients from successive image frames may be used to build a Kalman filter for recursively estimating the relative position and attitude of the helicopter with respect to the circle in camera coordinate. Because the measurements, Z, are nonlinear functions of the position vector (s), surface normal vector (n) and the radius of the circle, an extended Kalman filter must be used. The Kalman filter considered in this study have a linear continuous state model of the form

$$\dot{X} = F(t)X(t) + G(t)U(t) + \zeta_X(t) \tag{8}$$

where X is the state vector, U is the control input, ζ_x is a continuous white noise with covariance Q_c (representing modeling uncertainty), and F(t) and G(t) are time varying matrices. Using a sampling interval of ΔT s, (8) can be replaced by the discrete form

$$X(k+1) = \Phi(k)X(k) + \Gamma(k)U(k) + \zeta_x(k) \tag{9}$$

where $k \equiv i\Delta T$, $k+1 \equiv (i+1)\Delta T$, $i = 1,2,3,\cdots$, $\Phi(k)$ is the state transition matrix and $\Gamma(k)$ is the input distribution matrix. The matrices $\Phi(k)$ and $\Gamma(k)$ are computed numerically, except in special cases. The $\zeta_x(k)$ is a discrete white noise sequence with covariance $Q = Q_c/\Delta T$. The measurements $Z(t)$ are non-linearly related to the state through the vector function $h[X(t)]$ and can be linearized to give the measurement equation. Thus we have

$$Z(k)=h[X(k)]+\zeta_Z(k) \qquad (10)$$

Given the state equations (9) and the measurement of the ellipse coefficientz $Z(k)$ of (10), the estimate $X(k)$ and its error covariance matrix can be computed recursively using the Kalman filter.

The Kalman filter consists of two parts.

1) Measurement update: The measurement update is done whenever a new measurement is available. Prior to processing a new measurement $Z(k)$, we have the estimated value of the state X and the covariances $P(k)$, $Q(k)$, and $R(k)$. The new measurement improves the estimate of the state and its covariance. The updated values are

$$X(k)=X(k)^-+K(k)[Z(k)-h(X(k)^-)]$$
$$P(k)=[I-K(k)H(k)]P(k)^- \qquad (11)$$

where the matrix of partial derivatives

$$H(k)=\partial h(X)/\partial X \qquad (12)$$

and the Kalman filter gain $K(k)$ is computed using

$$K(k)=P(k)H^T(k)[H(k)P(k)H^T(k)+R(k)]^{-1} \qquad (13)$$

2) Time Update: This part of the filter accounts for the system dynamics and propagates the state and its covariance matrix until the next measurement is made.

$$X(k+1)^-=\Phi(k)X(k)+\Gamma(k)U(k)$$
$$P(k+1)^-=\Phi(k)P(k)\Phi(k)^T+\Gamma(k)Q(k)\Gamma(k)^T \qquad (14)$$

Here, we choose the relative position, s, and the attitude (surface normal vector), n of the helicopter with respect to the circle in camera coordinate and the circle radius as the state vector. Thus:

$$X\equiv(s_1,s_2,s_3,n_1,n_2,n_3,r)^T$$

Then the matrices in the state equation (9), the control input U and the noise are given as follows.

$$F(t)=\begin{bmatrix} 0 & -p_3 & p_2 & 0 & 0 & 0 & 0 \\ p_3 & 0 & -p_1 & 0 & 0 & 0 & 0 \\ -p_2 & p_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -p_3 & p_2 & 0 \\ 0 & 0 & 0 & p_3 & 0 & -p_1 & 0 \\ 0 & 0 & 0 & -p_2 & p_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G(t)=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$U\equiv\nu^T=(\nu_1,\nu_2,\nu_3)^T \qquad (15)$$
$$\zeta_x=(0,0,0,0,0,0,0)^T$$

where $(\nu_1,\nu_2,\nu_3)^T$ and $(p_1,p_2,p_3)^T$ are the linear and angular velocity of the helicopter in camera coordinate provided by the on-board sensors.

The conversion of the continuous time-varying state model (8) to the discrete model (9) is done assuming $F(t)$ and $G(t)$, and $U(t)$ to be constant over a small interval of time ΔT. Usually, this is done by numerical techniques, however by the above assumption we can derive the state transition matrix $\Phi(k)$ and the input distribution matrix $\Gamma(k)$ analytically. The detailed form of these matrices are given in Appendix.

4. EXPERIMENT ON SYNTHETIC DATA

The above estimation process was evaluated on the computer-generated measurements, 5

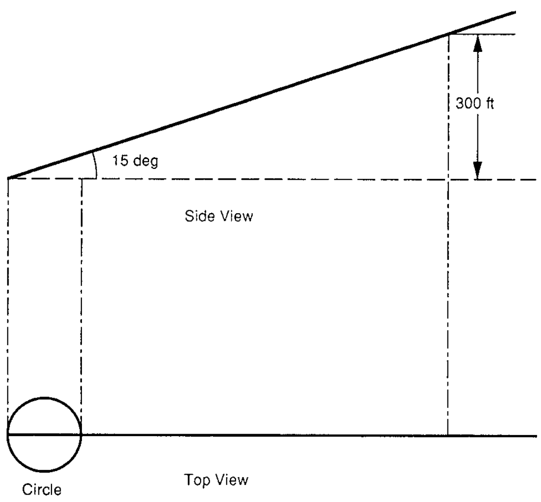


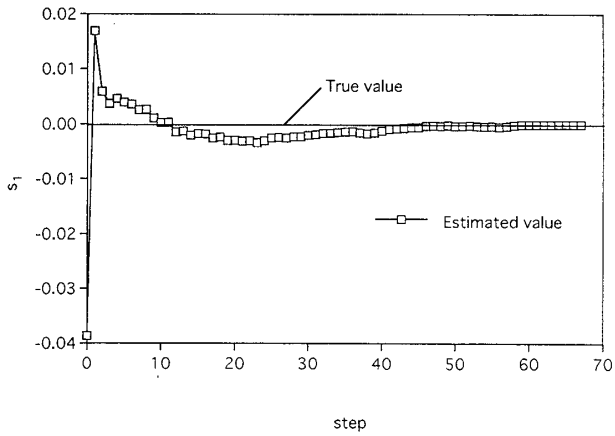
Fig. 3 Approach Path

coefficients of the ellipses.

The measurement values of 5 coefficients of the observed ellipse were calculated at each time step as follows: We first calculate the exact ellipse coefficients from the geometrical relationship between the helicopter and the circle with known camera parameters; then add the measurement error. The error is modeled by assuming that the position of the pixel on the observed ellipse is contaminated by zero-mean Gaussian noise with one (or two) pixel covariance.

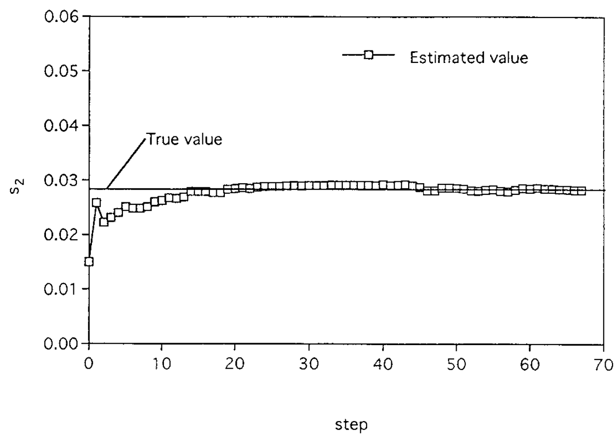
The assumed path of the helicopter is shown in Fig. 3. The constant glide slope approach is generally selected as the path for landing by visual. The estimation process is started when the helicopter reaches 300ft. altitude and we assume the velocity along the track to be constant.

Fig. 4 shows the typical convergence process of the 7 state variables, the position of the helicopter from the center of circle, the surface normal vector of the circle-plane in camera coordinate and the circle radius. Each component of the position vector and the circle radius are normalized by the altitude at which the task starts (300ft.). We can see that after about 40



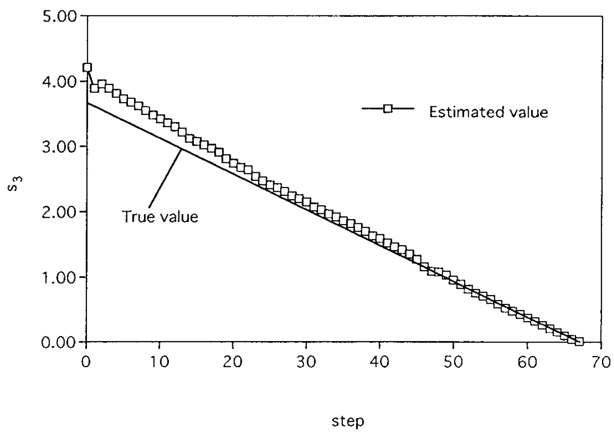
(a) x component of position vector, S_1

Fig. 4 True and estimated values of state vector



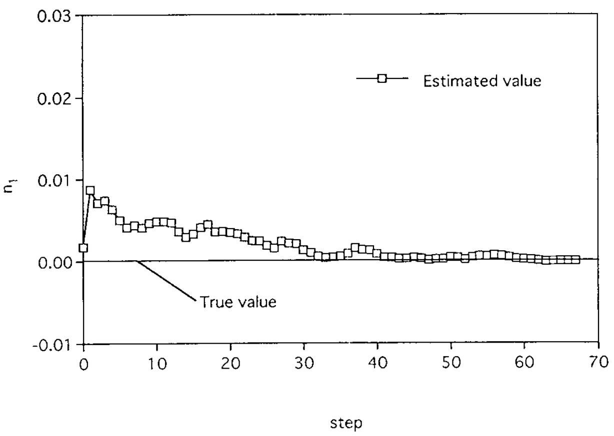
(b) y component of position vector, s_2

Fig. 4 True and estimated values of state vector



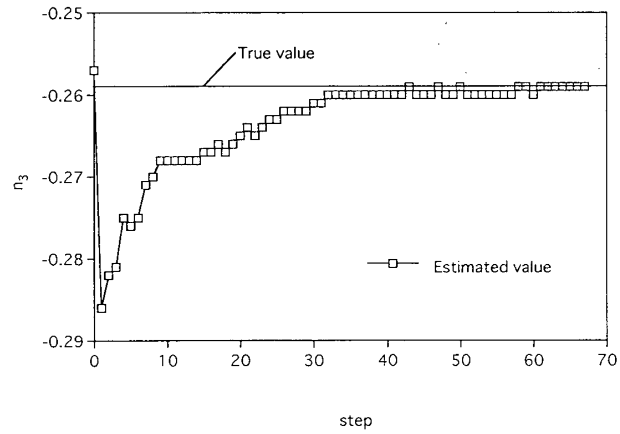
(c) z component of position vector, s_3

Fig. 4 True and estimated values of state vector



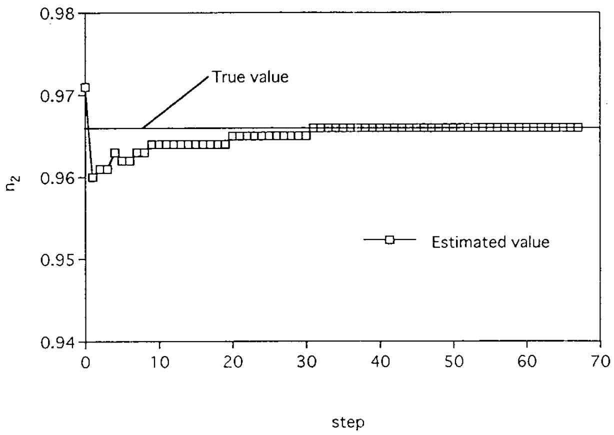
(d) x component of surface normal vector, n_1

Fig. 4 True and estimated values of state vector



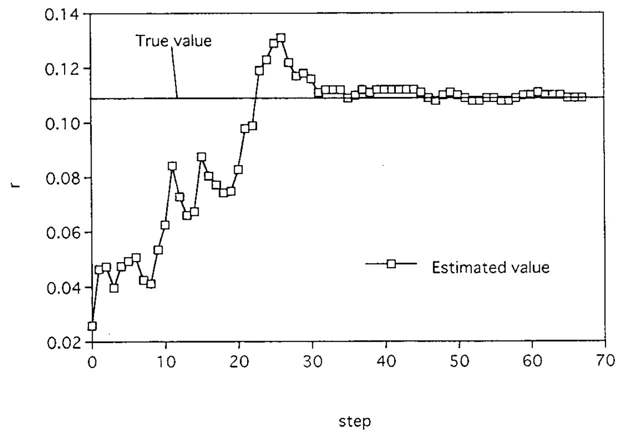
(f) z component of surface normal vector, n_3

Fig. 4 True and estimated values of state vector



(e) y component of surface normal vector, n_2

Fig. 4 True and estimated values of state vector



(g) radius of the circle

Fig. 4 True and estimated values of state vector

steps, the process gives good estimation for the variables. The deviation of the estimated values are within 2% of the exact value.

The numbers of steps required for the convergence under the various initial conditions are shown in Table 1. We assume that initial estimation errors of the state variables are at most 40% of the exact value and generated them by random process. Table 1 tells that the speed of convergence greatly depends on the initial state, although we have not found yet how it does. The effect of the time interval between time steps is also shown in this table. The smaller time interval requires more steps for convergence. This is because the assumed error model gives more accurate measurement as the helicopter approaches to the destination (the circle) and the process with the smaller time interval requires the more numbers of steps to reach the same position than that with larger time interval.

The effect of the measurement accuracy was investigated by using measurement error model constructed with position error of 2 pixel covariance. In Table 1 and 2, the case with the same number were computed with the same initial state. The more accuracy in the measurements doesn't always result in the faster convergence.

Table 1
Required Number of Steps for Convergence
(1 pixel error model)

Time Interval (sec)	1/6	1/12
Case 1	35	62
Case 2	41	71
Case 3	17	15
Case 4	21	37
Case 5	31	51

Table 2
Required Number of Steps for Convergence
(2 pixel error model)

Time Interval (sec)	1/6
Case 1	40
Case 2	10
Case 3	30
Case 4	40
Case 5	31

5. CONCLUSION

A method to estimate the relative position and attitude of a helicopter with respect to the heliport from image sequence taken by on-board camera was presented. The actual procedure for the estimation is

1) preprocessing (noise reduction, edge enhancement, etc.)
2) detection of the ellipse in the scene
3) calculation of 5 coefficients of the detected ellipse
4) estimation of the position, attitude, and the circle radius by the above method

at each time step. We have shown the recursive estimation process proposed here works well under the measurement error of pixel position on the circle modeled by a zero-mean Gaussian noise with a one (or two) pixel covariance. This model considers the error accompanied with procedure 1) and 2). There are the other factors that affect the measurement, such that the width of the marking and the vibration of the vehicle. These must be evaluated by working on the actual image data. We conducted the laboratory experiment by constructing the miniaturized environment and now work on the data.

The estimation process requires pretty a lot of steps for the convergence. However, this is not unnatural because the human pilot continuously updating their estimation by observing the ground scene.

APPENDIX

The state transition matrix Φ and the input distribution matrix Γ are given as follows when the linear and angular velocity of the helicopter are constant over a small interval of time ΔT. The derivation can be found in [3] or [6].

$$\Phi=\begin{bmatrix}[\phi] & [0] & \mathbf{0}\\ [0] & [\phi] & \mathbf{0}\\ \mathbf{0}^T & \mathbf{0}^T & 1\end{bmatrix} \qquad [\phi]=\begin{bmatrix}\phi_{11} & \phi_{12} & \phi_{13}\\ \phi_{21} & \phi_{22} & \phi_{23}\\ \phi_{31} & \phi_{32} & \phi_{33}\end{bmatrix} \tag{16}$$

$$\phi_{11}=ap_1^2+\cos(a\Delta T)$$
$$\phi_{12}=ap_1p_2-bp_3$$
$$\phi_{13}=ap_1p_3+bp_2$$
$$\phi_{21}=ap_1p_2+bp_3$$
$$\phi_{22}=ap_2^2+\cos(a\Delta T)$$
$$\phi_{23}=ap_2p_3-bp_1$$
$$\phi_{31}=ap_3p_1-bp_2$$
$$\phi_{32}=ap_2p_3+bp_1$$
$$\phi_{33}=ap_3^2+\cos(a\Delta T)$$

$$\Gamma=\begin{bmatrix}\gamma\\ [0]\\ \mathbf{0}^T\end{bmatrix} \qquad [\gamma]=\begin{bmatrix}\gamma_{11} & \gamma_{12} & \gamma_{13}\\ \gamma_{21} & \gamma_{22} & \gamma_{23}\\ \gamma_{31} & \gamma_{32} & \gamma_{33}\end{bmatrix} \tag{17}$$

$$\gamma_{11}=cp_1^2+\sin(a\Delta T)/a$$
$$\gamma_{12}=cp_1p_2-ap_3$$
$$\gamma_{13}=cp_3p_1+ap_2$$
$$\gamma_{21}=cp_1p_2+ap_3$$
$$\gamma_{22}=cp_2^2+\sin(a\Delta T)/a$$
$$\gamma_{23}=cp_2p_3-ap_1$$

$$\gamma_{31} = cp_3p_1 - ap_2$$
$$\gamma_{32} = cp_2p_3 + ap_1$$
$$\gamma_{33} = cp_3^2 + \sin(\alpha\Delta T)/\alpha$$

where

$$[0] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\alpha^2 = p_1^2 + p_2^2 + p_3^2$$
$$a = (1 - \cos(\alpha\Delta T))/\alpha^2$$
$$b = \sin(\alpha\Delta T)/\alpha$$
$$c = \Delta T/\alpha^2 - \sin(\alpha\Delta T)/\alpha^3$$

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