Synthesis of Quasi-Mountain Range Patterns and Its Display Techniques

Masayuki IIZUKA*

A recursive midpoint displacement method is often used in order to simulate a wide variety of quasi-mountains. In comparison with the results displayed by the above method, quasi-mountain range patterns with multipeak may be simply generated by a non-recursive method which uses a kind of the function of two variables and an amount of random altitude displacement. A contour map called topographical map and its 3-D perspective view are demonstrated from the viewpoint of synthesis of quasi-fractal mountains.

1. Introduction

Fractals (a word coined by Mandelbrot[1]) have blossomed tremendously in the past few years. Fractal geometry and its concepts have become central tools in most of the natural sciences. The observation of a geometry of nature by him have led us to think about clouds, mountains, forest, trees, river, etc. in a new scientific way. Fractal patterns and images appear complex, yet they often arise from simple stochastic and deterministic rules.

Simple and practical recursive midpoint subdivision techniques: ordinary midpoint displacement method and successively modified altitude displacement method are studied from the viewpoint of simulation of fractal relief images[2]. In the former, new altitude displacement at the midpoint of primitive parts in the x-y plane depends on altitude values at old grid points only in each subdivision process. Altitude displacement at all grid points in the latter is modified directly except for last altitude values in a prior step of midpoint subdivision.

In comparison with synthesis of quasi-mountain range patterns besed on the results of two types of the midpoint displacement method, quasi-mountain range patterns with multipeak may simply generated using a kind of two variable function and an amount of random infinitesimal change. The topographical map called a contour line diagram and its perspective view for fractal relief patterns are displayed in this study. Quasi-fractal perspective views generated by means of the two variable function techniques are compared with the results of the recursive midpoint subdivision techniques from the viewpoint of synthesis of quasi-mountain range patterns with multipeak. Fractal aspects of computer-generated relief images are discussed together with the role of typical simulation parameters such as dot resolution, threshold value, etc.

Department of Electronic Engineering
Tokyo Institute of Polytechnics
平成2年9月12日受理

2. Computer-Generated Relief Patterns

2.1 An Overview of Fractal Techniques

Fournier et al.[3] introduced a simplification of the fractal and stochastic method and proposed a recursive subdivision algorithm based on mathematical approximations of one dimensional Brownian motion.

N. M-Thalmann et al.[4] carried out a geometric study of parameters for the recursive midpoint subdivision in the case of both one dimensional direction models and two dimensional models using triangles. They demonstrated the effects of the three important parameters : threshold, eccentricity and displacement. As basic primitives for the recursive midpoint method, triangle, square, tetrahedron etc. have been usually used for generating fractal images.

Anjyo[5] proposed a simple spectral approach to stochastic modeling for various natural objects. This technique is carried out by means of a mathematical expression called power spectrum and a few kinds of specified parameters for simulation models such as mountains, clouds, wave etc. Computer graphics techniques for modeling fuzzy objects such as fire, water and clouds have been extensively studied by W. T. Reeves[6].

On the other hand, a Fourier filtering method based on FFT techniques is useful for a kind of fractional Brownian motion (fBm). Generated discrete samples, however, are periodic and annoying because of the nature of the Fourier transform. In this case, it is necessary to compute two or four times as many sample points as sctually needed and then to discard a part of the unnecessary sequence.

Recently, H. O. Peitgen and D. Saupe[7] edited a unique book "The Science of Fractal Images" which discusses fractals solely from the viewpoint of computer graphics. It contains many computer listings and basic algorithm for generating fractal patterns and objects.

In this study, quasi-mountain ranges with multipeak may be simply generated using a kind of two variable function and an amount of random infinitesimal change in comparison with the midpoint displacement method.

2.2 Midpoint Displacement Method[8]

A time function X(t) called fractional Brawnian Motion (fBm) may be defined using a normal (or Gauss) distribution function W(t) as follows.

$$X(t) = \int_{-\infty}^{t} W(s) \cdot ds \doteqdot \sum_{i} \{W_i(s) \cdot \varDelta s\} \tag{1}$$

where, W(t) : Normal (or Gauss) distribution function,
Random number with N $(m=0, \sigma=1)$

The above time function X(t) may be defined using a Gamma function in place of Eq. (1).

$$X(t) = \{1/\Gamma(H+0.5)\} \int_{-\infty}^{t} (t-s)^{H-0.5} W(s) \cdot ds \tag{2}$$

where, H : parameter relative to Fractal dimension,
$0<H<1$
Γ : Gamma function

Deviation of infinitesinal increments of Brownian noise: ΔX is treated quantitatively by the following expression.

$$\left.\begin{aligned} \mathrm{Var}\{X(t_i)-X(t_j)\} &= E[|X(t_i)-X(t_j)|^2] \\ &= K|t_i-t_j|^{2H} \\ &= \sigma^2\cdot(t_i-t_j)^{2H} \end{aligned}\right\} \tag{3}$$

where, Var $\{\cdot\}$: Deviation, E $[\cdot]$: Expectation,

$X(t)$: Brownian noise, σ : Standard deviation,

t_i, t_j : time K : constant

The midpont altitude: f_m between f_L and f_R may be estimated using the following expressin on condition that the altitude of both sides is f_L and f_R, respectively.

$$f_m = \{f_L+f_R\}/2 + \text{amount of fractal displacement} \tag{4}$$

where, Min $(f_L, f_R) \leqq fm \leqq$ Max (f_L, f_R) or $0 \leqq f_m$

Deviation of two variable function: $Z=f(x, y)$ is defined extending an idea of Eq. (3).

$$\left.\begin{aligned} \mathrm{Var}\{\varDelta Z\} &= E[|\varDelta Z|^2] \\ &= E[|f(x_i, y_i)-f(x_j, y_j)|^2] \\ &= \sigma^2\cdot d^{2H} \end{aligned}\right\} \tag{5}$$

where, d: distance, $d=\sqrt{(x_i-x_j)^2+(y_i-y_j)^2}$

In general, an amount of fractal displacement D is computed using the following expression.

$$\left.\begin{aligned} D &= \sqrt{\mathrm{Var}\{\varDelta Z\}}\cdot \mathrm{GAUSS} \\ &= \sigma\cdot d^H\cdot \mathrm{GAUSS} \\ &= K\cdot \mathrm{GAUSS} \end{aligned}\right\} \tag{6}$$

where, GAUSS: Normal distribution, N $(m=O, \sigma=1)$

According to the central limiting theorem, a numerical value of GAUSS parameter may be simply computed using the following expression.

$$\left.\begin{aligned} \mathrm{GAUSS} &= \{Y-E[Y]\}/\sqrt{\mathrm{Var}\{Y\}} \\ &= \sum_{I=1}^{n} Y_i-(\mathrm{Nrand}/2)\cdot P\}/\{\sqrt{\mathrm{Nrand}/12}\cdot P\} \\ &= \{(1/P)\cdot\sqrt{12/\mathrm{Nrand}}\cdot\sum_{I=1}^{n} Y_i\}-\sqrt{3\cdot \mathrm{Nrand}} \end{aligned}\right\} \tag{7}$$

where, Y_i : ith uniform pseudo-random number, $0 \leqq Y_i < 1$

P: Parameter of closed interval $[0, P]$,

(For example, $P=2^{15}-1$ or $2^{31}-1$)

n; Nrand: Total number of uniform pseudo-random number

$n=\mathrm{Nrand}$

2.3 Simulation of Fractal Patterns by Two Variable Function Techniques

In place of midpoint displacement techniques, the following two kinds of two variable functions are very useful for the simulation of quasi-fractal relief patterns[9].

$$
\left.\begin{aligned}
Z &= f_1(x, y) + f_2(x, y) + \text{amount of fractal displacement} \\
&= \sum_{l=1}^{m}\left\{\frac{A_l}{B_l\cdot(x-\alpha_l)^2 + C_l\cdot(y-\beta_l)^2 + D_l}\right\} \\
&\quad + \sum_{l=1}^{n}\{A_l\cdot\exp[-(B_l\cdot(x-\alpha_l)^2 + C_l\cdot(y-\beta_l)^2 + D_l)]\} \\
&\quad + E\cdot GAUSS
\end{aligned}\right\} \qquad (8)
$$

where, $Xmin \leqq x \leqq Xmax$, $Ymin \leqq y \leqq Ymax$,

$Zsh \leqq z \leqq Zmax$, Zsh: Threshold value

A_l, B_l, C_l, D_l: Positive constant parameter

α_l, β_l: Geometrical position value of single peak mountain

m ; n: Number of multipeak mountain

Though the second order polynominal $f_1(x, y)$ and another polynominal $f_2(x, y)$ with exponential function are different from each other in expression, they have alomst the same function by making an appropriate choice from the combination of positive constant parameters. It should be noted that if the value of two variable function Z is less than a certain threshold value Zsh, then Z is often replaced with zero level approximately. As a result, the boundary along the zero level may be more or less exaggerated in contrast to the value of arbitrary altitude $Z=f(x, y)$, when displaying the perspective relief patterns.

3. Computer Simulation of Fractal Relief Patterns

Fig. 1 (a) and (b) shows an example of the contour map and its perspective view displayed by means of recursive midpoint displacement techniques in the case of dot resolution: 33 x 33. The relief pattern with quasi-multipeak is demonstrated as a result of the combination of many parameters. Note that initial specified altitude values at four corner points are invariant (or constant "0") in each recursive step. Interactive and repeating techniques by trial and error are useful for generating certain desirable fractal relief patterns.

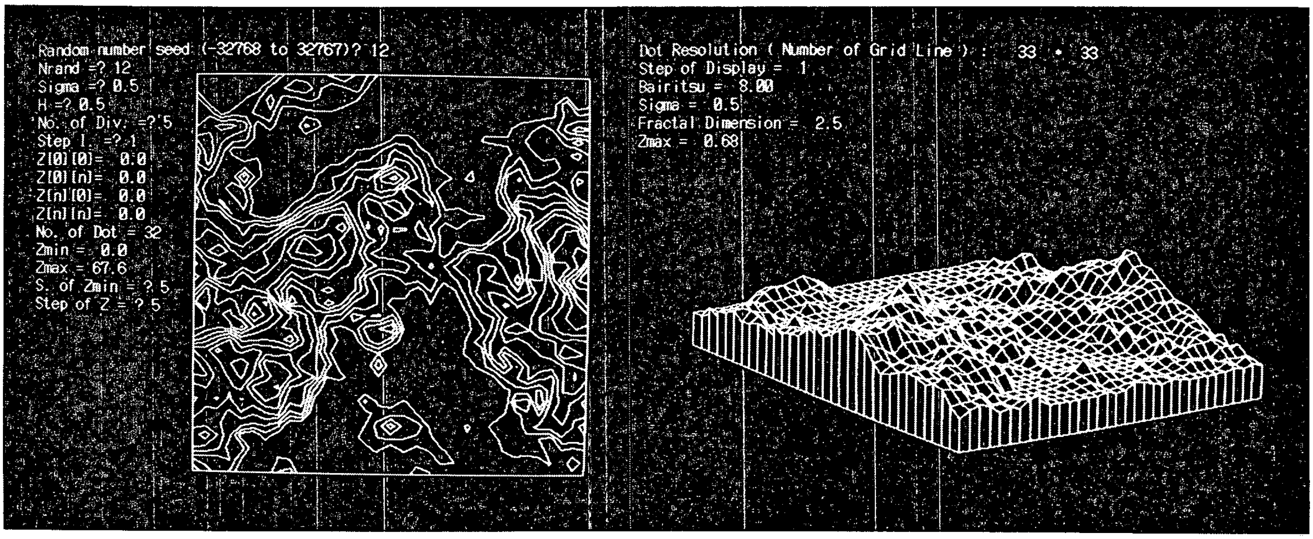

Fig. 1 Quasi-multipeak mountain range pattern in the case of dot resolution: 33×33
(a) Contour map (b) Perspective view

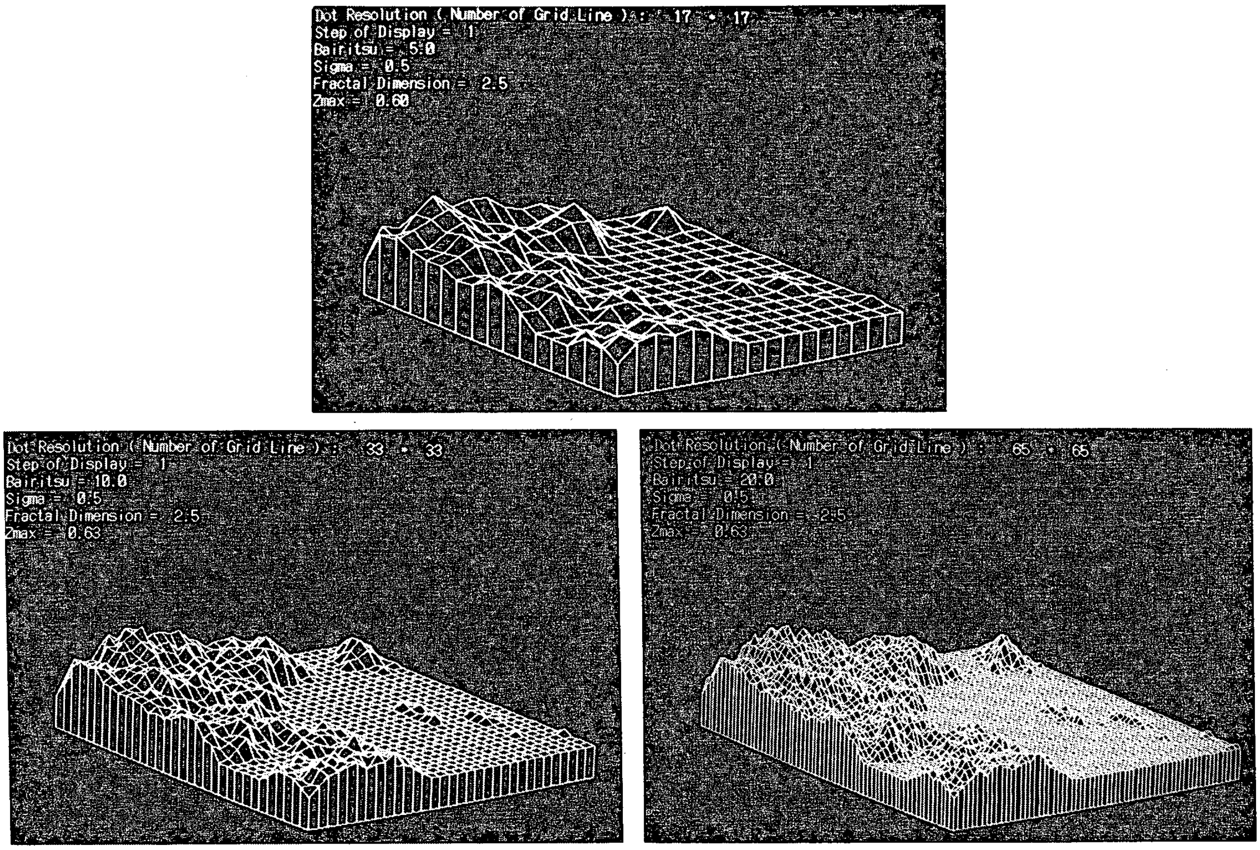

Fig. 2 Effect of dot resolution for mountain range patterns
(a) 17×17 (b) 33×33 (c) 65×65

Fig. 2 (a), (b) and (c) shows an example of the typical computation process of recursive midpoint subdivision techniques: ordinary midpoint displacement method in the case of fractal dimension: FD=2.5. Fractal relief images called perspective view are directly displayed from the computation results at the grid points. Owing to the effect of dot resolution, there is the difference of "Bairitsu" parameter among them, although an outline of the relief pattern is almost the same one another.

Fig. 3(a) and (b) shows a synthetic contour map and its perspective with quasi-multipeak by using the original dot resolution: 33×33. It is composed of four original relief patterns connected before and behind, besides right and left in the case of fractal dimension: 2.5. As result, the dot resolution of synthetic quasi-fractal relief pattern is asymmetric and extended to 97×65 approximately.

As a result of simulation, we obtain random and solitary patterns with some tops such as a kind of mountain range peaks. It should be noted that steep and uneven (rugged) fractal patterns may be demonstrated by using a threshold condition of altitude values. For example, a modified altitude: {f(x, y) +0.1}x "Bairitsu" may be used when displaying fractal relief patterns. If the modified altitude: Z=f(x, y) is less than 0.1, then Z is replaced with "0" level. There are a few unlikely discrepancies and subtle differences at the edge or border part with zero level when conjoining the original relief pattern. A series of altitude values along the border part are often changed into zero

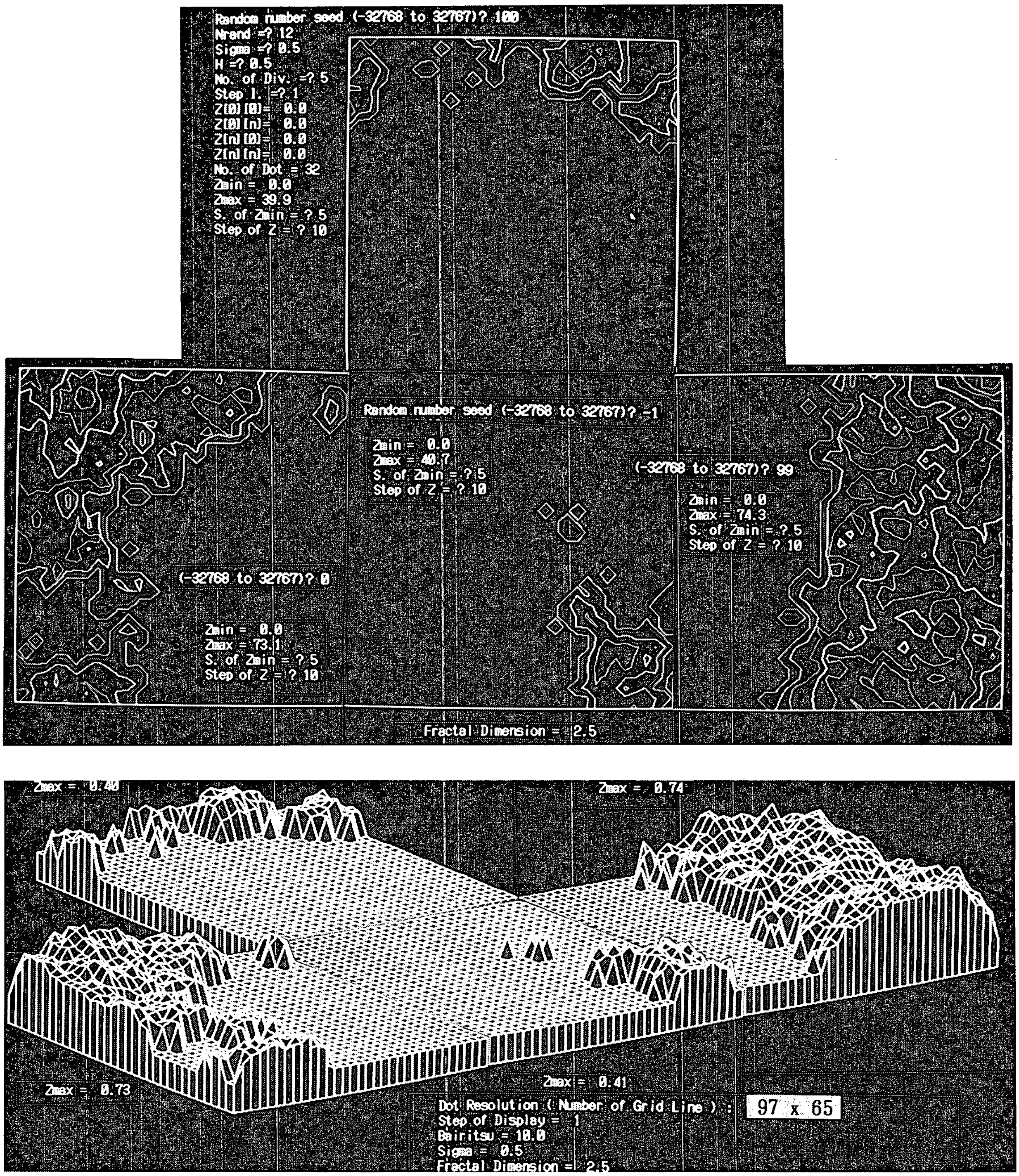

Fig. 3 Synthesis of quasi-multipeak mountain range pattern
(a) Quasi-contour map
(b) Quasi-perspective view

level intentionally because of the apparent and valid possibility of synthetic connection.

The visual appearance on an original or synthetic quasi-fractal perspective is cosiderably affected by the value of scaling factor called "Bairitsu" for display 3-D relief patterns on a CRT screen. It is very difficult to predict global aspects of fractal relief patterns in the recursive midpoint subdivision techniques in advance. Because fractal aspects of altitude displacement and its distribution, i. e., altitude values at each grid point are in disorder and quite distinct from between

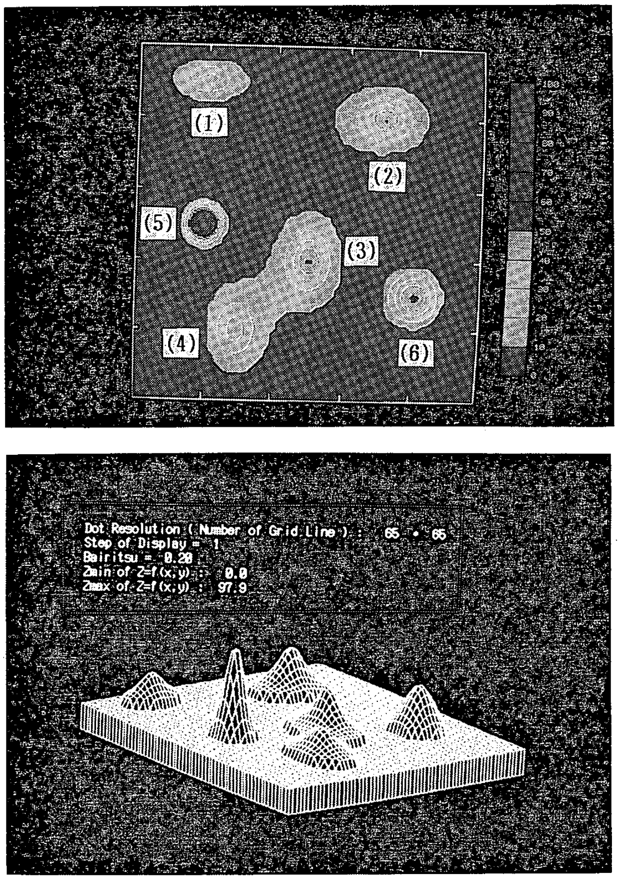

Fig. 4 Combination pattern of two types of two variable functions in the case of dot resolution: 65×65
(a) Contour map with color painting
(b) Perspective view

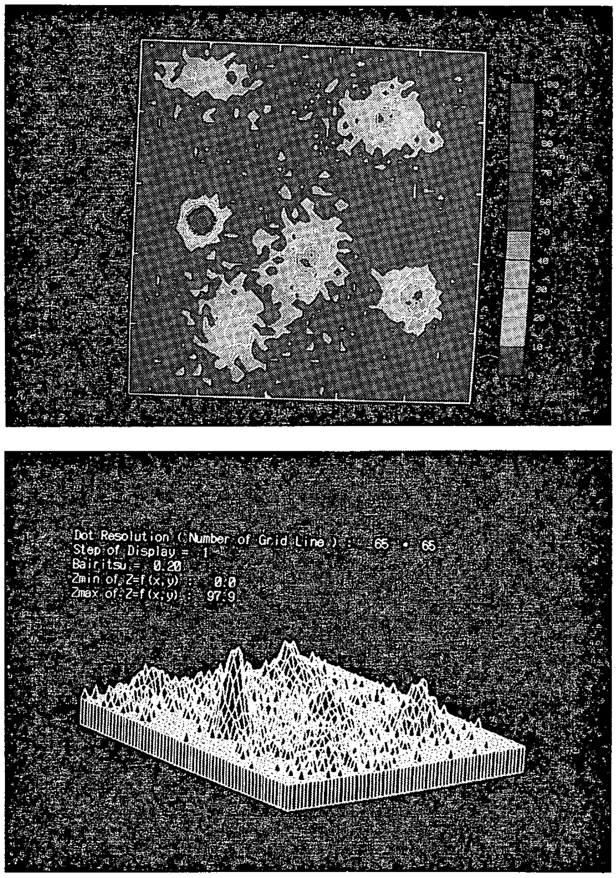

Fig. 5 Quasi-fractal multipeak mountain range pattern
(a) Contour map with color painting
(b) Perspective view

them, even though all the initial parameters are same (or invariant) except for the random number seed in the simulation algorithm.

Fig. 4 (a) and (b) shows a color painted contour map and its perspective view demonstrated by means of two variable function techniques in the case of dot resolution: 65×65. In this fugure, an amout of random altitude displacement at each grid point is not considered for the purpose of making the effect of two kinds of two variable functions clear. The notational numbers for six kinds of types corresponding to the parameters of Eq. (8) are specified as m=4 and n=2.

Note that a certain threshold condition, i. e., $Z_{sh}=10$ is used here. If $Z=f(x, y)$ is less than 10, then Z is replaced with zero level $Z=0$ purposely. As a result, the border parts between the non-zero altitude level and the zero level are enhanced.

Fig. 5 (a) and (b) shows the result of simulation with quasi-multipeak in contrast to Fig. 4. In this fugure, an amount of random altitude displacement at each grid point is considered for the purpose of making the fractal effect clear. The same threshold condition as well as the threshold condition in Fig. 4 is used here. Steep and uneven (rugged) quasi-fractal patterns are simply displayed without increasing the value of initial standard deviation and that of fractal dimension

Table 1 Typical parameters for two variable function in x-y domain : $0 \leqq x$ and $y \leqq 100$

	A	B	C	D	α	β
(1)	3000	2	6	90	20	90
(2)	4000	2	4	80	70	80
(3)	4000	4	2	50	40	80
(4)	4000	4	2	120	30	20
(5)	100	0.05	0.05	0	20	50
(6)	50	0.02	0.02	0	80	30

[Note] Constant parameter : E=0 in Fig. 4
: E=−5 in Fig. 5

defined as FD=3 - H, on condition that Eq.(8) is used for generating the quasi-fractal mountain range patterss.

Table 1 shows the values of typical constant paremeters which are used in Eq.(8), i. e., the expression for the function of two variables. The two parameters α and β specify the center of geometrical position at grid point, and the other four parameters have a close relation to the altitude value : $Z=f(x, y)$, and the spread of its distribution. The value of parameter E plays a role of the fractal Brownian motion, i. e., determinative aspect of random altitude displacement in connection with the "GAUSS". The specified domains of two variables such as x and y have a relation to each value of four kinds of constant parameters except for the parameter E.

4. Conclusions

To sum up, apparent and intuitive characteristics between the ordinary recursive midpoint displacement method and the method based on the function of two variables, i. e., the non-recursive method have been studied from the practical and demonstrative viewpoint for generating quasi fractal mountain range patterns.

(1) It is possible to display a large variety of fractal relief patterns by altering an initial random number seed and/or a Nrand parameter on the assumption that all the others are constant or fixed.

(2) The global aspects of 3-D fractal patterns depend on not only a few main parameters such as random number seed, initial standard deviation, fractal dimension, ect. but a scaling factor or a threshold condition for displaying a perspective view in the case of the ordinary recursive midpoint displacement method.

(3) The modified and synthesized quasi-fractal mountain range patterns with multipeak may be simply generated applying the two variable function method rather than applying the ordinary recursive method in connection with a kind of threshold condition for altitude values.

Acknowledgment

A computer program for displaying a set of contour lines was developed by S. Fukui and successively was modified by H. Ohshita and the author.

A practical program with hidden line elimination process for displaying $Z = f(x, y)$ was originally developed by H. Tamura on a FM-7 personal computer. The author would like to express his thanks to Y. Ohe and H. Ohshita for altering an algorithm of the program in part for a PC-9801 personal computer.

References

1) Benoit B. Mandelbrot: "Fractals: from, chance and dimension", WH Freeman, (1977); "The fractal geometry of nature", WH Freeman, (1982)
2) M. Iizuka and H. Ohshita: Mapping and Visualization of Computer-Generated Fractal Relief Patterns, Bulletin of Nagoya Institute of Technology, Vol. 41 (1989) 139-150
3) A. Fournier et al. : Computer rendering of stochastic models, Commun ACM, Vol. 26, No. 3 (1982) 371-384
4) N. Magnenat-Thalmann et al.: A geometric study of parameters for the recursive midpoint subdivision, The Visual Computer, Vol. 3, No. 3 (1987) 145-151
5) K. Anjyo: Extension Fractal based on choise of simple parameters for displaying mountain, cloud, wave etc., NIKKEI COMPUTER GRAPHICS No. 24 (1988) 132-140 [in Japanese]
6) W. T. Reeves: Particle systems - a technique for modeling a class of fuzzy objects, Proc. SIGGRAPH'83, Computer Graphics, Vol. 17, No. 3 (1983) 359-376
7) H. O. Peitgen and D. Saupe eds.: "The Science of FRACTAL IMAGES", Springer-Verlag (1988)
8) D. Saupe: Algorithms for random fractals, in "The Science of FRACTAL IMAGES" edited by H. O. Peitgen and D. Saupe, Springer-Varlag (1988)
9) S. Masumoto, T. Noto, K. Wadatsumi and K. Shiono: Color-Painted Contouring of Grid Data by Personal Computer, Information and Geology, No. 11 (1986) 177-186 [in Japanese]